Screening and Dissipation at the Superconductor-Insulator Transition Induced by a Metallic Ground Plane

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We study localization phenomena in two dimensional systems of charged particles in the presence of a metallic ground plane with a particular focus on the superconductor-insulator transition. The ground plane introduces a screening of the long-range Coulomb interaction, and provides a source of dissipation due to the gapless diffusive electrons. The interplay of these two effects leads to interesting physical phenomena which are analysed in detail in this paper. We argue that the generic superconductor-insulator transition of charged particles in the presence of the ground plane may be controlled by a *fixed line* with variable critical exponents. This is illustrated by an explicit calculation in an appropriate ϵ expansion. In contrast, the universal properties of the superconductor-Mott insulator transition in the clean limit at commensurate densities are shown to be unmodified by either the metal or the long-range Coulomb interaction. A similar fixed line can arise in the presence of a metallic ground plane for quantum Hall plateau transitions. Implications for experiments on Josephson-junction arrays and quantum Hall systems are described.

I. INTRODUCTION

Despite many decades of effort, the properties of matter in the vicinity of various transitions from conducting to insulating phases remains poorly understod. The best known example is the transition from a metal to an insulator. During the last few years, a number of such phenomena have been subjected to serious experimental study, particularly in low dimensional systems. These include transitions from a superconductor to an insulator [1], various transitions in two dimensional systems showing the quantum Hall effect [2], and an apparent transition from an as yet unidentified conducting phase to an insulator in two dimensional Silicon MOSFET's [3]. This increase in experimental activity has raised questions that require an improved theory of such localization phenomena.

In this paper, we study localization phenomena in two dimensional systems in the presence of a proximate ground plane formed by a diffusive two dimensional electron gas. Our focus will be the superconductor-insulator transition, though many of our results generalize to other transitions as well. Recent experiments have studied the effect of such a metallic plane on the quantum transition to superconductivity in Josephson junction arrays [4] and in homogenously disordered thin films in a magnetic field [5] . In these experiments, the actual tunneling of charged excitations between the superconductor and the metallic plane is very weak, and may be ignored. However, the charge carriers in the two systems are coupled together by the Coulomb interaction. Such a metallic plane has several effects. First, one might expect some screening of the long-ranged Coulomb interaction between the charges in the system undergoing the localization transition. Second, the coupling to the gapless excitations in the metal provides a source of "dissipation". The interplay of these two effects leads to interesting differences in the localization phenomena from that in the absence of the metallic plane. In particular, we argue that the zero temperature localization transition may be controlled by a *fixed line* with variable critical exponents.

The interest in studying the role of a metallic plane stems from several different directions. A particularly difficult issue for localization theory is the role of the long-range Coulomb interaction and its possible screening. For the integer quantum Hall transitions, for instance, while short-range interactions have been argued to be irrelevant at the non-interacting fixed point [6], the nature of the criticality with Coulomb interaction is very poorly understood. For metal-insulator or superconductor-insulator transitions, the Coulomb interaction is screened in the conducting phase but retains its long-range character in the insulating phase. This evolution in the screening properties of the system as it moves through the transition contributes to the difficulty in developing a theoretical understanding. Introducing a metallic plane enables exerting some control on the screening and may enable separating out phenomena that are specific to the long-range Coulomb interaction.

A different motivation comes from theoretical attempts to understand experiments on the destruction of superconductivity at zero temperature due to quantum effects. In situations where the transition is to an insulator, and at asymptotically low temperatures, a "boson-only" model that describes the localization of Cooper pairs while ignoring the fermionic degrees of freedom is expected to describe the universal critical properties [7]. However, in a number of experiments, the situation is different [1,5]. The non-superconducting phase may be a weakly localized metal, or in some cases ap-

pear to be truly metallic (with no trace of localization effects) in the temperature range currently probed in experiments. In these cases, a "boson-only" model is presumably inadequate, and it is necesary to include the underlying electrons. However a full treatment of the problem of Cooper pairs coupled to gapless electrons is prohibitively difficult—it therefore helps to study simpler situations where some if not all the effects are treated reliably. Such a simplification is provided by considering "boson-only" models coupled to gapless metallic electrons through density-density Coulomb interactions, in which the decay of the boson into pairs of electrons is ignored. Precisely this situation is realized in experiments on Josephson junction arrays where a proximate metallic plane is introduced [4]. Remarkably, as we show in this paper, such a coupling between the Cooper pairs and the metallic electrons has a profound effect on the properties of the transition.

The nature of the superconducting phase and its transition to the insulator in the presence of a metallic plane has been addressed before in the literature. We believe that these previous treatments miss several crucial aspects of the physics. Ref. [8] considered a model of a Josephson junction array with a short-ranged capacitance matrix that essentially amounts to assuming a *logarithmic* Coulomb interaction energy between the charges rather than the $1/r$ interaction that obtains for long distances and controls the universal properties of the transition. This difference profoundly modifies the physics—hence the results of Ref. [8] are not capable of describing the experiments of interest [4,5]. There are also a number of papers on "local dissipative" models, where the dissipation is claimed to arise from spatially localized excitations. In a short section below, we discuss the relationship between these models and our work. First in our work unlike in Ref. [9], the physical origin of the dissipation is clear - it is due to the gapless diffusive electrons in the metal. Furthermore, as we discuss in detail, the fixed line obtained here has an entirely *different* origin from the fixed line claimed to exist in the model studied in Ref. [9] (see also [10]). Deep inside the superconducting phase, the dynamics of the phase of the superconducting order parameter in the presence of the coupling to the metal was considered by Gaitonde [11], who argued that the plasmon was overdamped at long wavelengths. We derive the correct phase dynamics and show that in fact the plasmon mode survives at long wavelengths.

For most of this paper, we focus on the superconductor-insulator transition of repulsively interacting bosons in the presence of density-density coupling to a diffusive two dimensional electron gas. We begin by first considering the effects of the metallic plane on the superconducting phase. We show that the plasmon mode of the superconductor survives essentially unmodified. This is contrary to naive expectations based on the assumption that a short-ranged screened interaction between the bosons correctly describes *all* aspects of the physics with the metal present. We find that, because of their slow diffusive motion, the electrons in the metal are unable to screen out the faster charge fluctuations associated with the plasmon. This analysis sets the stage for a discussion of the general properties of the transition to the insulator. With the metallic plane present, we give a general argument that points to the existence of a *fixed line* controlling the transition. The fixed line is parametrized by the value of the conductivity of the metal. Universal critical properties will then be determined by the metal conductivity.

To substantiate our general arguments we present a detailed analysis of a specific model which is appropriate to describe the experiments in Ref. [4]. We consider the superconductor-insulator transition at commensurate density for bosons on a square lattice in the presence of the metal. We first provide crude estimates of the phase boundary and show that the presence of the metal stabilizes the superconducting phase relative to the insulator. We derive an effective action for the Cooper pair degrees of freedom by integrating out the diffusive metallic electrons which is then used to discuss the critical phenomena at the transition. In the clean system, we argue that neither the metal nor the long-ranged part of the Coulomb interaction affects the critical properties so that the transition is in the universality class of the $D = 2 + 1$ XY model. This result is strongly suggested by two separate calculations—a large-N generalization of our effective model, and an ϵ expansion of an appropriate generalization to dimensions other than 2. However moving away from this idealized limit is expected to alter the critical properties and produce the fixed line expected from the general arguments advanced earlier. We illustrate this by including disorder which nevertheless preserves the particle/hole symmetry of the commensurate problem. A double ϵ expansion [12] along the lines of Ref. [13] is then performed, including both the Coulomb interaction and the effect of the metal, which indeed finds the expected fixed line.

The generality of our arguments for the existence of a fixed line in the presence of the metal is then illustrated in the context of other concrete yet tractable models of localization transitions. Specifically, we show that a model for the quantum Hall transition considered in Ref. [14] should also possesses this feature. Indeed we expect that the generic quantum Hall transition with long-range Coulomb interactions could, in the presence of a proximate metal, also be controlled by a fixed line with variable critical exponents.

The implications of our results for experiments on Josephson-junction arrays, superconducting films, and quantum Hall transitions are considered in Section VII. An important qualitative lesson is that the presence of a proximate metallic plane *does not* by itself guarantee that the universality class is that of the short-ranged prob-

lem. Under what circumstances then is the latter realized? For the generic superconductor-insulator or quantum Hall transition, our analysis indicates that when the fixed line scenario is realized, and the conductivity of the metal is very large (compared to e^2/h) there will be a region near the transition where the universal behaviour is controlled by the short-range fixed point. Very close to the critical point however there will be a crossover to behaviour controlled by the appropriate point on the fixed line. The presence of a fixed line directly impacts experiments seeking to measure universal transport at these two dimensional localization transitions. Indeed, a temperature-independent conductivity will obtain right at the transition point, with a value that will vary along the fixed line and hence may appear to be non-universal. However, this conductivity will be universally related to the critical exponents (which too will vary along the fixed line). Experimental realization of the fixed line may be easier to observe for the quantum hall transition than for the superconductor-insulator transition as the universal scaling regime appears to be more easily accesible in the former.

It is interesting to speculate on the implications of our results for the general superconductor-metal transition. Our analysis shows that a simple Coulomb coupling between the order parameter (boson) degrees of freedom and gapless diffusive electrons already leads to profound modifications of the universal properties from that of a boson-only model. To correctly describe the superconductor-metal transition , it is necessary to include processes where the Cooper pairs can decay into a pair of electrons and vice versa into the model. If any portion of the fixed line survives this inclusion, then the conductivity at the transition will again appear to be non-universal and will be determined by the value of the marginal coupling at the point on the fixed line that controls the transition. We note that experiments on two-dimensional superconductors undergoing a transition to a weakly localized metal do see a temperature-independent but apparently non-universal conductivity at the transition.

II. GENERAL ARGUMENTS

We begin with some simple observations on the effects of a density-density coupling between the system of interest and a diffusive 2DEG. For concreteness, we consider the physical situation shown in Figure 1. A two-dimensional Josephson junction array (JJA) is separated by an insulating slab from a two dimensional electron gas (2DEG), with conductivity σ_e. The Cooper pairs in the JJA couple to the electrons in the 2DEG solely via the Coulomb interaction. We assume that the electron motion in the 2DEG is diffusive, and ignore weak localization effects. For the experimental situation of interest this is well justified since the typical 2DEG conductivities are large enough that weak localization effects are insignificant at currently accessible temperatures.

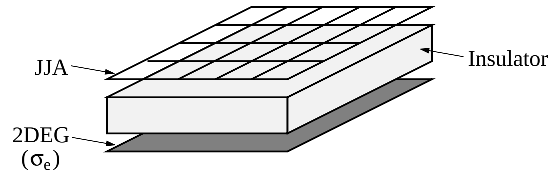

FIG. 1. The setup considered in this paper, based on the experiments of Rimberg et al.[4] A Josephson junction array (JJA) is separated from a two dimensional electron gas (with conductivity σ_e that can be tuned) by an insulating layer that blocks tunneling of electrons. The Cooper pairs in the JJA therefore interact with the electrons in the 2DEG solely via the Coulomb interaction.

A. Superconducting Phase

Consider first the effect of the 2DEG on the properties of the superconducting phase. In the absence of the 2DEG, the superconducting state sustains a "plasmon" excitation associated with oscillations of the phase of the superconducting order parameter. This mode is gapless in two dimensions with energy $\sim \sqrt{k}$ at low momenta k. This is in contrast to the linear dispersion of the sound mode obtained in *neutral* superfluid systems (*i.e* with short-ranged interactions). Upon coupling the charged superconductor to the 2DEG, should we expect to recover the linear dispersing mode due to the screening of the long-ranged part of the Coulomb interaction? As shown below, the answer is no. On the time scale relevant to the plasma oscillations, the slow diffusive motion of the electrons in the 2DEG is unable to screen out the Coulomb interaction. Thus the plasmon survives essentially unmodified.

To establish this result, note that in the superconducting state, the phase of the Cooper pairs can be treated as a classical variable. We can then write down the classical equations of motion for the phase $\phi(r,t)$ which obeys the Josephson relations:

$$\partial_t \phi(r,t) = 2eU(r,t) \tag{1}$$

$$-2e\rho_s \nabla\phi(r,t) = \vec{J}_c \tag{2}$$

where $\vec{J}_s$ is the supercurrent, $U(r,t)$ is the potential and ρ_s is the superfluid stiffness (assumed constant). From charge conservation in the JJA layer, we have (ρ_c is the Cooper pair density):

$$\partial_t \rho_c + \vec{\nabla} \cdot \vec{J}_c = 0 \tag{3}$$

The potential $U(r,t)$ is generated by both the Cooper pair charge density as well as the electronic charge density in the 2DEG (ρ_e):

$$U(r,t) = \int d^2r' V(|r-r'|)(\rho_c(r',t) + \rho_e(r',t)) \quad (4)$$

$$(5)$$

where the interaction at large distances falls off as $V(|r-r'|) \sim \frac{1}{|r-r'|}$ and, for convenience, we have neglected the small but finite vertical separation between the JJA and the 2DEG.

Finally, to completely describe the dynamics of this system, we need the equations of motions that govern charge motion in the 2DEG:

$$J_e(r,t) = -\sigma_e \nabla U(r,t) - \mathcal{D}\nabla\rho_e \quad (6)$$

$$0 = \partial_t \rho_e + \nabla \cdot J_e \quad (7)$$

where $\mathcal{D}$ is the diffusion constant for electrons, and J_e is the current density in the 2DEG. From equations (1,3,4,6), we obtain the equation of motion for the phase variable, which is most conveniently written in terms of its Fourier components $\tilde{\phi}(q,\omega)$ defined by:

$$\tilde{\phi}(k,\omega) = \int d^2r dt e^{-ik\cdot r} e^{i\omega t}\phi(r,t). \quad (8)$$

The equation of motion is:

$$[\frac{\omega^2}{(2e)^2}\frac{1}{\tilde{V}_{eff}} - \rho_s k^2]\tilde{\phi}(k,\omega) = 0 \quad (9)$$

where the effect of the 2DEG is simply to replace the Coulomb interaction by the effective interaction $\tilde{V}_{eff}$:

$$\frac{1}{\tilde{V}_{eff}(k,\omega)} = \frac{1}{\tilde{V}(k)} + \frac{\sigma_e k^2}{-i\omega + \mathcal{D}k^2} \quad (10)$$

The Fourier transform of the $1/r$ Coulomb interaction in two dimensions $\tilde{V}(k)$ takes the form $\tilde{V}(k) = \frac{2\pi}{|k|}$ in the long wavelength limit.

In the absence of the 2DEG ($\sigma_e = 0$), we obtain the usual two-dimensional plasmon mode:

$$\omega_{pl}(k) = \sqrt{(2e)^2\rho_s\tilde{V}(k)k^2} = \sqrt{8\pi e^2\rho_s}\sqrt{k} \quad (11)$$

Notice that with the 2DEG present, in the *static* limit $\omega = 0$, the Coulomb interaction is screened:

$$\tilde{V}_{eff}(k,\omega=0) = 1/(\frac{|k|}{2\pi} + \frac{\sigma_e}{\mathcal{D}}) \quad (12)$$

For situations that are not static, we cannot automatically assume this screened Coulomb form for the interaction, rather, the dynamics of the 2DEG will have to be taken into consideration.

The plasma mode is obtained by solving:

$$\frac{\omega(k)^2}{(2e)^2}\frac{1}{\tilde{V}_{eff}(k,\omega(k))} - \rho_s k^2 = 0$$

which, in the long wavelength (k small) limit reduces to:

$$\omega(k) = \omega_{pl}(k) - i(\pi\sigma_e)k \quad (13)$$

Now, since the oscillation frequency of this mode goes as the square root of the wave vector, it is much larger (in the long wavelength limit) than the damping rate which depends linearly on wave vector . Thus, as claimed above, the plasmon is a well defined excitation and retains its $\sqrt{k}$ dispersion despite the presence of the 2DEG.

This problem of the phase dynamics deep inside the superconducting state in the presence of a metallic gate, was examined earlier in [11] where, in contast to the present work, the plasmon was found to be overdamped at long wavelengths. We believe this result to be in error; in particular, the form of the dielectric function used there (equation (20a) of reference [11]) is valid only in three dimensions and not applicable to the two dimensional geometry of the current problem.

B. Nature of the Phase Transition

We now consider the nature of the zero temperature phase transition between the superconducting and insulating states of the JJA. In this section we present some general arguments that strongly suggest the presence of a fixed line of critical points controlling the generic (including disorder) superconductor-insulator transition of charged particles in the presence of the metallic plane. Note that in the presence of quenched disorder the superconductor to insulator transition will be a continuous transition in two dimensions [15].

Consider a system of bosons in two dimensions interacting via a $1/r$ Coulomb potential and coupled to a diffusive 2DEG in the manner described. Such a system can be described in terms of a charged bosonic field minimally coupled to the time component of a gauge field (A_0). The spatial components of this gauge field can be ignored since we are in the extreme type II limit, at zero external field. Here, we will examine the action governing the gauge field, which takes the following form in imaginary time (a detailed derivation is in the next section):

$$S_A = \frac{1}{2}\int đ^2k đ\omega\, V_{eff}^{-1}(k,\omega)|\tilde{A}_0(k,\omega)|^2 \quad (14)$$

$$= \frac{1}{2e^{*2}}\int đ^2k đ\omega\, \frac{|k|}{2\pi}|\tilde{A}_0(k,\omega)|^2 + \frac{\sigma_e}{2e^{*2}}\int đ^2k đ\omega\, \frac{k^2}{|\omega| + Dk^2}|\tilde{A}_0(k,\omega)|^2 \quad (15)$$

where $e^* = 2e$ is the charge of the Cooper pairs, and we have introduced the compact notation $đ\omega = \frac{d\omega}{2\pi}$.

The first term on the right hand side of equation (15) gives rise to the Coulomb interaction while the second term arises from the coupling to the diffusive 2DEG. Notice that these terms contain non-analytic functions of

k,ω. If we perfom a Renormalization Group transformation on this system, by integrating out the boson and gauge field modes that lie within an energy-momentum shell of finite thickness, then additional terms of this form *cannot* be generated. Hence the scaling of these operators arises solely from rescaling of space-time and of the fields. Since the rescaling of the gauge field is fixed by the requirement of gauge invariance, the scaling dimensions of these operators can be easily derived. Gauge invariance requires that the gauge field A_0 must scale as the inverse time, to preserve the minimal coupling form of the action. Hence its Fourier transform in two dimensions, $\tilde{A}_0$, scales as:

$$[\tilde{A}_0(k,\omega)] = \mathbf{L}^2 \tag{16}$$

where $\mathbf{L}$ has dimensions of length. Thus the Coulomb term has dimensions:

$$[\int d^2k d\omega |k||\tilde{A}_0(k,\omega)|^2] = \mathbf{L}^{1-z} \tag{17}$$

where z is the dynamical critical exponent ([time] $= \mathbf{L}^z$). Thus, the coupling strength of the Coulomb interaction e^{*2} has the renormalization group flow:

$$\frac{de^{*2}}{dln(b)} = e^{*2}(z-1) \tag{18}$$

when modes between $[\Lambda, \Lambda/b]$ are integrated out. Thus, a fixed point that has a finite value of the Coulomb coupling necessarily also has $z = 1$ [7]. This can also be seen from the following simple argument. The $1/r$ Coulomb interaction in two dimensions, is non-analytic in Fourier space. Therefore, the inverse linear power law of the interaction cannot be renormalized. In addition, if the fixed point has a finite value of the Coulomb charge, then the energy must scale as $1/r$ which immediately implies that the dynamical critical exponent is unity.

We can now apply a similar analysis to the second term in equation (15). In the long wavelength limit, the $\mathcal{D}k^2$ term in the denominator is irrelevant, so long as the dynamical critical exponent for the theory satisfies $z < 2$. This condition will need to be checked at the end of the calculation to ensure consistency. Using the known scaling dimension of the gauge field, we find the interesting result:

$$[\int d^2k d\omega \frac{k^2}{|\omega|}|\tilde{A}_0(k,\omega)|^2] = \mathbf{L}^0 \tag{19}$$

Not only is this operator marginal from its scaling dimension, it is always strictly marginal! This is an exact statement, arising from gauge invariance and the non-analytic form of the gauge field action, and is *not* just the engineering (tree level) dimension of this operator.

The presence of a strictly marginal operator in the fixed point action will in general lead to a line of fixed points. This conclusion should obtain if the renormalized Coulomb charge remains non-zero at the transition. Universal critical properties (such as exponents) will vary continuously along this line, and hence will depend on σ_e. As argued below, the Coulomb charge is expected to be non-zero for the generic superconductor-insulator transition that occurs in the presence of disorder, from a superconducting phase to a gapless Bose-glass phase. This points to the existence of a fixed line in the presence of the 2DEG.

In the absence of the metallic gate, the argument for a finite value of the Coulomb charge at the superconductor to Bose-glass transition was reported in [7]. There, the singularity in the long wavelength behaviour of the compressibility (arising from the Coulomb interaction) on both sides of the transition was used to argue that the Coulomb charge remains finite at the transition itself. Although this argument fails in the presence of the metal due to screening in the static limit, a different argument can be made in this case to obtain information about the generic transition. Imagine that at this transition, the Coulomb charge does not take on a finite value, but instead flows to zero. In this limit the gauge field will decouple from the bosons, which then only experience short-ranged interactions. The transition will therefore be in the dirty boson universality class for which it is known that $z = 2$ [16]. However, this scenario would not be consistent with the flow equation (18) which indicates that near this fixed point the Coulomb charge grows and renders it unstable. Hence we are led to conclude that the Coulomb charge will remain non-zero at the transition between the Bose-glass and the superconductor. This leads to two possible scenarios. In the first scenario, the Coulomb charge takes on a finite value at the transition, as in the case without the metal. Then, a line of fixed points all with $z = 1$ will obtain. In the second scenario, the Coulomb charge flows away to infinity, and the gauge fluctuations are completely controlled by the second term on the right hand side of equation (15). Such fixed points could be stable for $z > 1$, and once again we are lead to expect a line of fixed points, but with $z > 1$. For consistency, we will also require $z < 2$ to justify our assumption in deriving (19) where the term containing the diffusivity which is quadratic in the momentum was neglected in comparison to the frequency. The question of $z \geq 2$ fixed points in the presence of the metallic 2DEG will not be discussed in the present work, and is left for future study.

Further support for a line of fixed points is provided by our calculations on a model of the superconductor-insulator transition with particle-hole symmetric disorder, where these scenarios are explicitly realised. The critical properties of this model are explored within a double epsilon expansion, and both a line of fixed points with $z = 1$, as well as a separate line of fixed points with $z > 1$ are found, corresponding to the two scenarios

described above.

The situation in the clean problem, where the transition is between the superconductor and the Mott insulator is, however, different. For this special case, despite the presence of the strictly marginal operator, none of the universal quantities are affected, since the Coulomb charge flows to zero at the transition, which is hence controlled by a single (3D XY) critical point. We will nevertheless consider this problem first, since it will allow us to erect our formalism in a technically more simple case. The properties of this model are then considered within the large-N and ϵ expansions. Subsequently, we consider a modified model, that includes particle-hole symmetric disorder, which behaves more like the generic case. This model is studied using the ϵ,ϵ_τ expansion [12] [13] which reveals that indeed the universal properties of the transition depend on the value of the 2DEG conductivity σ_e, and hence a fixed line results.

III. SUPERCONDUCTOR-INSULATOR TRANSITION IN THE CLEAN LIMIT

In this section we describe a model of an array of Josephson junctions coupled to a diffusive 2DEG via Coulomb interactions. We ignore the presence of disorder and assume that the junctions are at integer Cooper-pair filling. (Note that in the clean limit, at non-integer filling, the insulating phase would generally break translational symmetry - a direct transition from the superconductor to such an insulator is expected to be first order).

Consider an array of superconducting islands at positions labelled by $\vec{r}$. We model each of these islands as an $O(2)$ quantum rotor; this is a legitimate approximation when there are on average many Cooper pairs present on each island. The deviation of the boson charge from the background value is denoted by n_r. Then, the number of Cooper pairs on each site $\frac{n_r}{e^*}$ is conjugate to the phase on each island ϕ_r.

$$[\frac{n_r}{e^*}, e^{i\phi_r}] = e^{i\phi_r} \tag{20}$$

Coupling the charge fluctuations in the JJA layer to the diffusive 2DEG via Coulomb interactions as described earlier, we obtain:

$$\begin{aligned}\hat{H} = &-E_J \sum_{<rr'>} \cos(\phi_r - \phi_{r'}) \\ &+\frac{1}{2}\sum_{rr'}(n_r + \delta n_e(r))V(r-r')(n_{r'} + \delta n_e(r')) \\ &+H^0_{2DEG}\end{aligned} \tag{21}$$

where δn_e is the fluctuation of electronic charge density in the 2DEG, the Coulomb interaction at large distances is taken to be $V(r-r') \sim 1/2\pi|r-r'|$ (the factor of 2π is introduced for later convenience). H^0_{2DEG} is the Hamiltonian of the diffusive 2DEG, without the Coulomb interaction between electrons. We now recast this as a path intergal over the bosonic (n,ϕ) and fermionic (ψ_e) degrees of freedom. It is also convenient to introduce a Hubbard-Stratonovich decoupling of the long-range interaction using an auxilliary field A_0. (As will be clear, A_0 may be interpreted as the scalar component of a gauge field.) This leads to the following representation for the finite temperature ($= 1/\beta$) partition function:

$$\mathcal{Z} = \int \mathcal{D}\phi\mathcal{D}n\mathcal{D}A_0\mathcal{D}\psi_e e^{-S} \tag{22}$$

where the phase satisfies the boundary conditions $\phi_r(\beta) = \phi_r(0) + 2\pi m_r$ with m_r any integer, and

$$S = S_B + S_{int} + S_A[A_0] + S^0_{2DEG}[\psi_e] \tag{23}$$

$$\begin{aligned}S_B = &\int_\tau [\frac{i}{e^*}\sum_r n_r(\tau)\partial_\tau\phi_r(\tau) \\ &-E_J\sum_{<rr'>}\cos(\phi_r(\tau) - \phi_{r'}(\tau))]\end{aligned} \tag{24}$$

$$S_{int} = \frac{i}{e^*}\int_\tau \sum_r A_0(r,\tau)(n_r(\tau) + \delta n_e(r,\tau)) \tag{25}$$

$$S_A = \frac{1}{2e^{*2}}\int_\tau \sum_{rr'} A_0(r,\tau)V^{-1}(r-r')A_0(r',\tau). \tag{26}$$

Here S^0_{2DEG} is the action correspoding to the diffusive 2DEG in the absence of the Coulomb interaction, $\int_\tau = \int_0^\beta d\tau$ and V^{-1} implies the matrix inverse of the potential. Following the pioneering work of Hertz [17] on quantum phase transitions in fermionic systems, we integrate out the diffusive electrons in the 2DEG to derive an effective action for the bosons. The contribution arising from this proceedure is:

$$\begin{aligned}\mathcal{Z}[A_0] &= \int \mathcal{D}\psi_e e^{-S^0_{2DEG}} e^{-\frac{i}{e^*}\int_\tau \sum_r A_0(n_r(\tau)+\delta n(r,\tau))} && (27)\\ &= \mathcal{Z}[0] < e^{\frac{i}{e^*}\int_\tau \sum_r A_0(n+\delta n)} >_{2DEG} && (28)\end{aligned}$$

Performing a cumulant expansion for the average, and taking into account $<\delta n(r,\tau)>_{2DEG} = 0$ we have:

$$\begin{aligned}&< \exp\frac{i}{e^*}\int\sum_r A_0(n+\delta n) >_{S^0_{2DEG}} = \\ &\exp -\frac{1}{2e^{*2}}\int_{\tau,\tau'}\sum_{r,r'} <\delta n_r(\tau)\delta n_{r'}(\tau')>_{S^0_{2DEG}} A_0(r,\tau)A_0(r',\tau') \\ &+ ...\end{aligned} \tag{29}$$

As in Ref. [17], we shall neglect the three and higher body interaction terms that are generated to obtain:

$$\mathcal{Z}[A_0] \cong e^{-\frac{1}{2}\int_{\tau,\tau'}\sum_{r,r'}\Pi^0(r-r',\tau-\tau')A_0(r,\tau)A_0(r',\tau')} \tag{30}$$

where

$$\Pi^0(r-r',\tau-\tau') = \frac{1}{e^{*2}} < \delta n_r(\tau)\delta n_{r'}(\tau') >_{2DEG} . \quad (31)$$

We thus obtain:

$$\mathcal{Z} = \int \mathcal{D}\phi \mathcal{D}n\mathcal{D}A_0 \mathcal{D}e^{-S_{eff}}$$

$$S_{eff} = \int_\tau \frac{i}{e^*}\sum_r n_r(\partial_\tau\phi + A_0) - E_J\sum_{rr'}\cos(\phi_r - \phi'_r) \quad (32)$$

$$+\frac{1}{2e^{*2}}\int_{\tau\tau'} A_0(r,\tau)V^{-1}_{eff}(r-r',\tau-\tau')A_0(r',\tau') \quad (33)$$

where

$$V^{-1}_{eff}(r,\tau) = V^{-1}(r)\delta(\tau) + \Pi^0(r,\tau). \quad (34)$$

For a diffusive metal the Fourier transform of Π^0 is given by:

$$\tilde{\Pi}^0(k,\omega) = \sigma_e\frac{k^2}{|\omega| + Dk^2} \quad (35)$$

where σ_e is the conductivity and $\mathcal{D}$ is the diffusivity of the metal. Thus, the Fourier transform of the effective potential is:

$$\frac{1}{\tilde{V}_{eff}(k,\omega)} = \frac{1}{\tilde{V}(k)} + \sigma_e\frac{k^2}{|\omega| + Dk^2} \quad (36)$$

$$= |k| + \sigma_e\frac{k^2}{|\omega| + Dk^2} \quad (37)$$

Note that in the static ($\omega = 0$) limit, the Coulomb interaction is screened as can be seen in equation (12). However, the question of screening is more subtle when it comes to dynamical phenomena (as we have seen in the case of the plasmon).

A. Shift in Phase Boundary

In this section we study the dependence of the zero temperature superconductor-insulator phase transition point on the 2DEG conductivity. Let us call the critical Josephson coupling E^c_J; for $E_J > E^c_J$ the system is in the superconducting phase. We would like to find the dependence of E^c_J on the 2DEG parameters. A rough estimate of this phase boundary is obtained by starting out in the superconducting phase and applying an analogue of the Lindemann criterion; i.e. we ask for what value of the parameters are the superconductor phase fluctuations of O(1).

It is convenient for this analysis to rewrite the partition function for the sytem in terms of the phase variable:

$$\mathcal{Z} = \int \mathcal{D}\phi \exp -\frac{1}{2}\int d\!\!{}^-{}^2k d\!\!{}^-\omega\, \frac{\omega^2}{e^{*2}}[\tilde{V}^{-1}_{eff}(k,\omega)]|\tilde{\phi}_{k\omega}|^2$$

$$+ E_J\int_\tau\sum_{rr'}\cos(\phi_r - \phi_{r'}) \quad (38)$$

with the bounary conditions $\phi_r(\beta) = \phi_r(0) + 2\pi m_r$, where m_r is integer. In the superconducting state, we can replace the cosine in the action (38) by a quadratic gradient term. The quadratic action for the phase fluctuations in the superconductor is:

$$S_{sc} = \int d\!\!{}^-{}^2k d\!\!{}^-\omega\, [\frac{\omega^2}{e^{*2}}\tilde{V}^{-1}_{eff}(k,\omega) + (E_Ja^2)k^2]|\tilde{\phi}_{k\omega}| \quad (39)$$

where a is the microscopic cutoff, and the integral over modes in momentum space is restricted to $|k| < \sqrt{4\pi}/a$. This combined with the 'Lindemann' condition ($\mathcal{A}$ is a constant of O(1)):

$$< \phi^2 >= \mathcal{A} \cong O(1)$$

leads to the follwing implicit equation for the critical point E^c_J:

$$\mathcal{A} = \int d\!\!{}^-{}^2k d\!\!{}^-\omega\, (\frac{\omega^2}{e^{*2}}[|k| + \sigma_e\frac{k^2}{|\omega| + \mathcal{D}k^2}] + E_Ja^2k^2)^{-1} \quad (40)$$

Inspecting the integral it is clear that the superconductor is stabilized in the presence of the 2DEG, i.e. the phase transition point $E^c_J(\sigma_e)$ is reduced compared to its value in the absence of the 2DEG: $E^c_J(\sigma_e) < E^c_J(0)$. This is schematically depicted in Figure (2). Qualitatively, this effect is to be expected, since the screening arising from the 2DEG contributes to weakening the Coulomb interaction, and hence enhances the stability of the superconduting state. Also, it has been argued [18] that coupling the phase fluctuations to external degrees of freedom ('dissipation') leads to reduced phase fluctuations which would also stabilize the superconducting phase. This effect has been observed in the experiments of Rimberg et al. [4] who were able to cross the insulator to superconductor phase boundary just by increasing the conductivity of the 2DEG. Experiments on Josephson junction arrays with chromium shunt resistors (that do not go superconducting), have been reported in [20]. A phenomenological model of that system can be written down which, in the superconducting phase, would be exactly the same action as in equation (39) above without the $\mathcal{D}k^2$ term, with the conductance σ_e being the conductance of the chromium resistor. In that experiment too, it was found that increasing the value of σ_e increases the stability of the superconducting region.

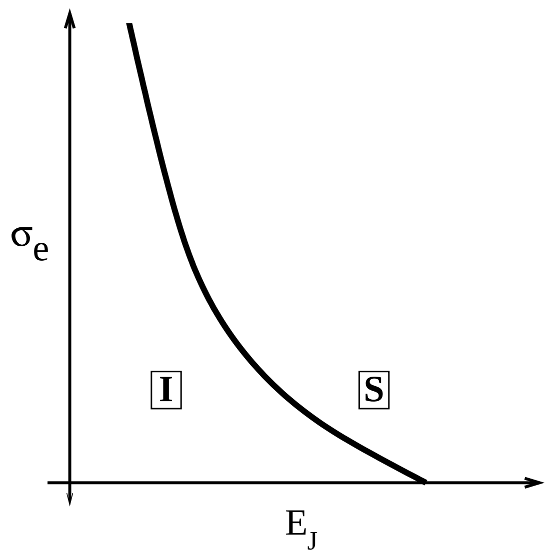

FIG. 2. The Superconductor-Insulator phase boundary in the presence of a 2DEG (conductivity σ_e). The 2DEG stabilizes the superconductor, as described in the text. Thus, the critical value of the intergrain Josephson coupling E_J at which superconductivity is established, decreases with increasing σ_e.

B. Critical Properties: Clean Case

In order to study the critical properties of this system, it is convenient to reformulate the action in equation (38), by rewriting it as a "soft spin" model, where, after coarse graining, the rotor "spin" $e^{-i\phi}$ is replaced by a complex field ψ. This automatically takes into account the nontrivial boundary conditions on the phase $\phi(\beta)$. The finite temperature partition function can then be written as a path integral over these fields:

$$\mathcal{Z} = \int \mathcal{D}\psi\mathcal{D}A_0 e^{-S_\psi - S_A^{eff}}$$
$$S_\psi = \int d^2r d\tau \frac{1}{c}|(\partial_\tau - iA_0)\psi|^2 + c|\nabla\psi|^2 + r|\psi|^2 + \frac{u}{4}|\psi|^4$$
$$S_A^{eff} = \frac{1}{2e^{*2}}\int d\!\!{}^-{}^2k d\!\!{}^-\omega\, [|k| + \sigma_e\frac{k^2}{|\omega| + \mathcal{D}k^2}]|A_0(k,\omega)|^2 \quad (41)$$

Since we are interested in the critical properties of this model, we can safely ignore the $\mathcal{D}k^2$ part in the denominator of the second term in (41). Also, we will scale out the bare sound wave velocity c that appears in the action above by a suitable rescaling of the imaginary time. All velocities will therefore be measured in units of this bare velocity. For convenience, we continue to use the same symbols for the scaled quantities, but these factors will have to be restored when comparing with the experiments.

In this section we consider the effect of the 2DEG and the Coulomb interaction on the properties of the superconductor-insulator transition in the *absence* of disorder. This is studied within two different approximations - the large-N technique and the ϵ expansion. As shown below, in both these approximation schemes, the Coulomb interaction is found to be marginally irrelevant, even in the presence of the metal. As a result, the gauge field decouples from the bosonic degrees of freedom at the critical point, and the presence of the strictly marginal operator (the coupling to the 2DEG) does not affect the critical properties of the system. The transition is therefore controlled by the same fixed point as in the clean case with short-range interaction and without any coupling to the metal. The agreement between the large-N and ϵ expansions is strong evidence that neither the Coulomb interaction or the coupling to the $2DEG$ affects the critical properties in the clean commensurate superconductor-insulator transition.

In the generic case, away from this idealized limit, we however expect that the combination of the Coulomb interaction and the coupling to the $2DEG$ will lead to a fixed line controlling the transition. This is illustrated in the next section by introducing disorder into the model.

1. Large N Calculation:

The effect of the 2DEG on the critical properties of the clean problem can be considered within the large-N approximation. We remain in $D = 2$ spatial dimensions, but generalize the model of a single species of boson to one that has N flavours of complex boson fields ψ_i:

$$S_\psi = \int d^2r d\tau \sum_{i=1}^{N} |(\partial_\tau - iA_0)\psi_i|^2 + |\nabla\psi_i|^2 + r_0|\psi_i|^2 + \frac{u}{N}[\sum_i |\psi_i|^2]^2 \quad (42)$$

$$S_A = \frac{N}{2e^{*2}}\int d\!\!{}^-{}^2k d\!\!{}^-\omega\, [|k| + \sigma_e\frac{k^2}{|\omega|}]|\tilde{A}_0(k,\omega)|^2 \quad (43)$$

As argued previously, the entire renormalization of the Coulomb charge e^* comes from rescaling the fields and the momenta. This leads to the following exact equation:

$$\frac{de^{*2}}{d\ln b} = (z-1)e^{*2} \quad (44)$$

while the term containing σ_e is exactly marginal. Here, we will derive an expression for the dynamical exponent (z) at the critical point - within a large-N approximation.

In the $N \to \infty$ limit, the (saddle point) solution to the above problem has $A_0 = 0$, and the critical point, at this order, is identical to that of the short-ranged model. We therefore need to go to next order in $1/N$ to see the effect of the gauge field coupling . In calculating z to order $1/N$ at the critical point, we only need to consider the self energy diagram shown in figure (5a) but within this approximation the gauge propagator $\mathcal{G}_A$ is replaced by its RPA form:

$$\frac{1}{N}\mathcal{G}_A^{-1}(k,\omega) = e^{*-2}[|k| + \sigma_e \frac{k^2}{|\omega|}] + \frac{k^2}{16\sqrt{k^2+\omega^2}} \quad (45)$$

as shown in Figure 5c. Therefore we find the dynamical exponent is given by:

$$z - 1 = \frac{1}{N} I(e^{*2}, \sigma_e) \quad (46)$$
$$I(e^{*2}, \sigma_e) = \int_0^{\pi/2} \frac{d\theta}{2\pi^2} \frac{\sin^2\theta(1-3\sin^2\theta)}{e^{*-2}(1+\sigma_e \tan\theta) + \frac{\sin\theta}{16}}$$

(Details of this derivation are relegated to Appendix A). In the absence of the metal and for small Coulomb charge we find: $I(e^* \to 0, \sigma_e = 0) = -\frac{5}{32\pi}e^{*2}$, and implies that a weak Coulomb interaction is marginally irrelevant at the short-ranged fixed point in the absence of the metal. Even in the presence of the metal, it is found that $I(e^{*2}, \sigma_e) < 0$, and hence the Coulomb interaction continues to be marginally irrelevent in the large-N approximation. Thus, the only fixed point solution to the flow equation (44) is $e_c^* = 0$. At this fixed point, the gauge field coupling may be ignored and the presence of the metal does not affect any of the universal critical properties. Therefore in the large-N approximation despite the presence of the metal, the transition is in the universality class of the short-ranged model.

2. The ϵ Expansion for the Clean Problem

In the dimensionality of interest $D = 2$, the quartic term $|\psi|^4$ is relevant by power counting. One way to control the RG flows is via the standard technique of considering the problem close to its upper critical dimension, the dimension at which the quartic coupling is also marginal by power counting ($D = 3$). Therefore, we write the effective action (41) in $D = 3 - \epsilon$ dimensions as:

$$S = S_\psi + S_A \quad (47)$$
$$S_\psi = \int d^D r d\tau |(\partial_t - iA_0)\psi|^2 + |\nabla\psi|^2 + r|\psi|^2 + \frac{u}{4}|\psi|^4 \quad (48)$$
$$S_A = \frac{1}{2e^{*2}} \int đ^D k đ\omega (|k|^{D-1} + \sigma_e \frac{k^D}{|\omega|})|\tilde{A}_0(k,\omega)|^2 \quad (49)$$

Notice that the action for the gauge field is written in a general dimension D, so as to always produce the $1/r$ form of the Coulomb interaction (if $\sigma_e = 0$). The term containing σ_e is continued to general dimensions so that it remains an exactly marginal operator.

This problem, in the absence of the metal ($\sigma_e = 0$) was considered in [21] where the Coulomb interaction was found to be marginally irrelevant. Here too we will investigate the flow of the Coulomb coupling, but in the presence of the metal. For $D < 3$, the non-analyticity of the Coulomb term in momentum space once again ensures that it is not renormalized by terms that are generated on integrating out high energy modes. Thus equation (44) again obtains for the flow of e^* while the term containing σ_e is exactly marginal. Here, we will calculate z to first order in ϵ, the relevant diagram is shown in Figure 7a. We find (see Appendix B 1 for details):

$$z - 1 = -\frac{e^{*2}}{6\pi^2} A(\sigma_e) \quad (50)$$
$$A(\sigma_e) = \frac{2}{\pi} \int_0^{\pi/2} d\theta \frac{\sin^2\theta(4\sin^2\theta - 1)}{1 + \sigma_e \tan\theta} \quad (51)$$

where $A(\sigma_e)$ is normalized so that $A(0) = 1$. In that limit, $\sigma_e = 0$, we recover the flow equation of Ref. [21], so that the Coulomb interaction is marginally irrelevant since the right hand side of (44) is negative. Even for non zero σ_e, it can be shown that $A(\sigma_e) > 0$ and thus the Coulomb interaction is marginally irrelevant despite the presence of the metal. The fixed point value of the Coulomb interaction then is $e_c^* = 0$. As a result, the gauge field is decoupled from the bosons at the critical point, and the presence of the strictly marginal operator does not affect the universal critical properties.

IV. LINE OF FIXED POINTS IN A MODEL WITH DISORDER

In this section, we present evidence to support our claim that the critical properties for the generic transition (as opposed to the idealized clean commensurate limit in Section III above) are controlled by a line of fixed points. We study a model of the disordered superconductor-insulator transition in the presence of Coulomb interactions and a metallic plane. This is in contrast to the zero disorder transition considered above. (Note that even in the absence of the metal the clean commensurate superconductor-insulator transition with Coulomb interactions has behaviour different from the generic case). We consider instead a slightly more realistic model that contains a special form of disorder that preserves the particle hole symmetry of the problem. In the absence of the metal, such a model was recently studied by Herbut [13] who found a $z = 1$ fixed point with a finite value of the Coulomb charge. Here we add the coupling to the metal, and first demonstrate that *a line of fixed points* with a finite value of the Coulomb charge and $z = 1$ can be obtained. Subsequently, a line of fixed points with $2 > z > 1$, for which the Coulomb charge flows to infinity, is also obtained.

The model we consider is essentially the one in equation (47) but with a coupling to disorder $\mathcal{V}$, included:

$$S = S_\psi + S_A + S_{dis} \quad (52)$$
$$S_\psi = \int d^D r d\tau (|(\partial_t - iA_0)\psi|^2 + |\nabla\psi|^2 + r|\psi|^2 + \frac{u}{4}|\psi|^4)$$

$$S_A = \frac{1}{2e^{*2}} \int đ^D k đ\omega (|k|^{D-1} + \sigma_e \frac{k^D}{|\omega|}) |\tilde{A}_0(k,\omega)|^2$$
$$S_{dis} = \int d^D r d\tau \mathcal{V}(r) |\psi(r,\tau)|^2$$

Notice, that the coupling to disorder preserves the particle hole symmetry of the model (the action is invariant under the particle-hole transformation: $\psi \to \psi^*$, $A_0 \to -A_0$). For quenched disorder, the random potential $\mathcal{V}$ is independent of imaginary time, and therefore has a big effect on the physics of the system. In the following, it is convenient to generalize it to a random variable that is correlated in ϵ_τ dimensions; in other words we assume the following statistics for the random potential:

$$< \mathcal{V} > = 0 \tag{53}$$
$$< \mathcal{V}(\mathcal{R}) \mathcal{V}(\mathcal{R}') > = \bar{W} \delta^{D+1-\epsilon_\tau}(R - R') \tag{54}$$

where the label R runs over both space and time components. For quenched disorder $\epsilon_\tau = 1$. However, it will be convenient to study the model in the limit of small ϵ_τ which will allow one to control the effects of disorder. This approximation was introduced in Ref. [12] to study the problem of bosons in the presence of disorder and short-ranged interactions.

In order to perform the disorder average, we use the standard replica technique. We introduce n copies of the fields $\psi \to \psi_\alpha$, $\alpha = 1 \dots n$, average over different realizations of the disorder potential, and finally take the $n \to 0$ limit. Averaging over disorder generates the following term which has a non-local interaction in ϵ_τ dimensions between the replica fields.

$$S = \sum_{\alpha=1}^{n} S_B[\psi_\alpha] + S_A + S'_{dis} \tag{55}$$
$$S'_{dis} = -\frac{\bar{W}}{2} \sum_{\alpha,\beta=1}^{n} \int d^{D+1}R d^{D+1}R' \delta^{D+1-\epsilon_\tau}(R-R') |\psi_\alpha(R)|^2 |\psi_\beta(R')|^2 \tag{56}$$

We now consider performing an RG transformation on the above action, by integrating out the short wavelength modes $1/\Lambda < a < b/\Lambda$, and then rescaling momenta, frequency and fields to obtain the flow equations for the various couplings. The calculation is controlled by working in $D = 3 - \epsilon$ space dimensions, and as mentioned, with disorder that is correlated in ϵ_τ dimensions. We assume that ϵ,ϵ_τ are small and derive the flow equations to lowest order in these quantities. Details of this derivation are relegated to Appendix B. After a redefinition of the coupling constants:

$$q^2 = \frac{e^{*2}}{2\pi^2}$$
$$\lambda = \frac{U}{8\pi^2}$$
$$W = \frac{\bar{W}}{2\pi^2} \tag{57}$$

the flow equations take the following form:

$$\frac{dq^2}{d\ln b} = (z-1)q^2 = (\frac{W}{8} - \frac{A}{3}q^2)q^2 \tag{58}$$
$$\frac{d\lambda}{d\ln b} = \epsilon\lambda + \frac{B}{2}\lambda q^2 - \frac{5}{2}\lambda^2 - \frac{C}{4}q^4 + \frac{11}{8}\lambda W \tag{59}$$
$$\frac{dW}{d\ln b} = (\epsilon + \epsilon_\tau)W + \frac{7}{8}W^2 + \frac{B}{2}q^2 W - 2\lambda W \tag{60}$$

and the coupling constant $\frac{1}{g} = \frac{\sigma_e}{e^{*2}}$ is strictly marginal and does not flow. The parameters A,B,C that appear in the flow equations are functions of σ_e and have been normalized so that $A(0) = B(0) = C(0) = 1$. They are given by the following expression:

$$A(\sigma_e) = \frac{2}{\pi} \int_0^{\pi/2} d\theta \sin^2\theta (4\sin^2\theta - 1) f(\sigma_e, \theta) \tag{61}$$
$$B(\sigma_e) = \frac{4}{\pi} \int_0^{\pi/2} d\theta \sin^2\theta f(\sigma_e, \theta) \tag{62}$$
$$C(\sigma_e) = \frac{4}{\pi} \int_0^{\pi/2} d\theta \sin^2\theta f^2(\sigma_e, \theta) \tag{63}$$

where

$$f(\sigma_e, \theta) = \frac{1}{1 + \sigma_e \tan\theta} \tag{64}$$

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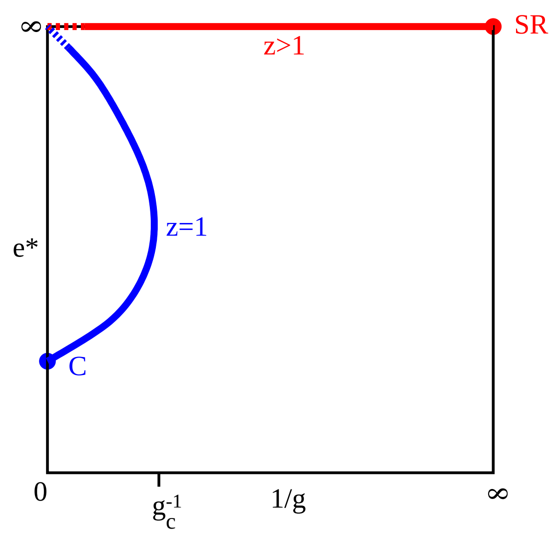

FIG. 3. The results of the ϵ, ϵ_τ expansion showing the fixed lines obtained in the presence of the marginal coupling $1/g$ which is proportional to the 2DEG conductivity. Only the fixed point value of the Coulomb charge e^* is plotted. The coupling constants e^* and g are measured in units of ϵ, where $\epsilon_\tau = \epsilon$ has been assumed. The $z = 1$ fixed points are obtained for $g^{-1} < g_c^{-1}$, while $z > 1$ fixed points are obtained for all g. The fixed points with both e^* and g large are not to be trusted since gauge field fluctuations are big in that case, and are shown with dashed lines. The limiting cases shown as C and SR, correspond to the Coulomb fixed point found in [13] and the short-ranged fixed point found in [12]

A. Fixed line with $z = 1$

We now solve for the fixed points of the flow equations above, assuming that the Coulomb charge e^* takes on a finite value at the transition. The coupling contants at the fixed point are proportional to ϵ but depend both on the ratio ϵ_τ/ϵ and on σ_e. In the following we will assume $\epsilon_\tau/\epsilon = 1$, a choice that is consistent with our aim to study the $\epsilon_\tau = \epsilon = 1$ limit. We evaluate the functions A,B,C numerically for various values of σ_e and find the fixed point values of the couplings which are quantities of order ϵ that depend on the value of σ_e. It turns out that fixed point solutions of this kind exist for σ_e in the range $0 < \sigma_e < 1.99$. While it is convenient to solve for the fixed points in terms of σ_e, they are more appropriately labelled by the value of the marginal coupling g, which can be obtained from the relation: $\frac{1}{g} = \frac{\sigma_e}{2\pi^2 q_c^2}$. When expressed in terms of this variable, the fixed points exist in the range $\frac{1}{g} < \frac{1}{g_c} \approx (572\epsilon)^{-1}$. In fact, as shown in figure 3, for every value of g in this range, there are a pair of fixed points. The fixed points with a smaller value of the Coulomb charge, evolve in the limit $\frac{1}{g} \to 0$ (which corresponds to removing the metallic plane) to the fixed point studied by Herbut in [13].

All of the fixed points obtained are found to be stable within a linear stability analysis; although the eigevalues are found to be complex, they have negative real parts. Thus we have a stable line of fixed points for a range of the 2DEG conductivity. The dynamical critical exponent z for all of these fixed points is unity. Other critical exponents can be expressed in terms of the fixed point couplings:

$$\eta = \epsilon q_c^2(\sigma_e) B(\sigma_e) \tag{65}$$

while the exponent ν can be expressed as:

$$\nu = \frac{1}{2} + \frac{1}{4}(\lambda_c(\sigma_e) - \frac{D(\sigma_e)}{12} q_c^2(\sigma_e) - \frac{W_c(\sigma_e)}{4}) \tag{66}$$

where the function $D(\sigma_e)$ (which has been normalized to be unity in the absence of the 2DEG, $D(0) = 1$) is given by:

$$D(\sigma_e) = \frac{4}{\pi} \int_0^{\pi/2} d\theta \sin^2 2\theta f(\sigma_e, \theta) \tag{67}$$

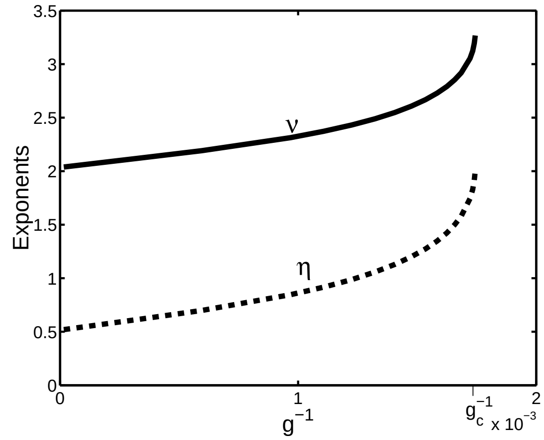

FIG. 4. The exponents η and ν for the $z = 1$ fixed line (lower branch in Figure 3) controlling the S-I phase transition. Along the horizontal axis is plotted the marginal coupling $1/g$ which is proportional to the 2DEG conductivity. For this plot we have set $\epsilon = \epsilon_\tau = 1$. From the dependence of the exponents on the conductivity it is clear that the critical properties are significantly affected by the presence of the 2DEG.

The remaining exponents can all be derived from a knowledge of these, if hyperscaling is assumed. The Chayes inequality $\nu \geq 2/D$ is satisfied by all these fixed points. A plot of these two critical exponents as a function of the marginal coupling $1/g$ (which is proportional to the 2DEG conductivity) is shown in Figure (4). Only the exponents for the lower branch of the $z = 1$ fixed line in figure 3 are shown. The exponent ν is found to increase with increasing 2DEG conductivity. If this qualitative effect that is present in the ϵ expansion persists unchanged in the physical problem, then this would predict that the critical region for a finite sample would increase as the 2DEG conductivity increases, which could be experimentally accessible. We return to this point in the following section.

This calculation therefore, provides us with an explicit demonstration of the general claims of this paper - that in the presence of the Coulomb interaction and a metal, the dirty superconductor to insulator transition can be controlled controlled by a line of fixed points, parametrized by the metal conductivity.

B. Fixed Line With $z > 1$

We now turn to the question of whether there are any stable fixed points in this model with $e^* = \infty$. For clarity we rewrite the gauge action as:

$$S_A = \frac{1}{2e^{*2}} \int d\!\!\bar{}^{\,D}k d\!\!\bar{}\,\omega\, |k|^{D-1} |\tilde{A}_0(k,\omega)|^2 + \frac{1}{2g} \int d\!\!\bar{}^{\,D}k d\!\!\bar{}\,\omega\, \frac{k^D}{|\omega|} |\tilde{A}_0(k,\omega)|^2 \tag{68}$$

where again g has beens used to denote the coupling strength of the marginal term, and is related to the bare couplings by $g = e^{*2}/\sigma_e$. Under the Renormalization Group transformation, g does not flow. Now, if the renormalized Coulomb charge were to flow to infinite strength, then the gauge fluctuations would entirely be controlled by the second term in the gauge action in equation (68). In order that this is a stable fixed point we need to ensure that $z > 1$, so that $\beta(e^*) > 0$ at the fixed point. In this limit, the flow equations can be obtained from the expressions (58 - 64) by formally taking the limit e^*, $\sigma_e \to \infty$ but holding the ratio e^{*2}/σ_e finite and equal to g. Defining:

$$\bar{g} = \frac{g}{2\pi^2}$$

yields the following flow equations:

$$\frac{d\lambda}{d\ln b} = \epsilon\lambda + \frac{1}{\pi}\lambda\bar{g} - \frac{5}{2}\lambda^2 - \frac{1}{4}\bar{g}^2 + \frac{11}{8}\lambda W \tag{69}$$

$$\frac{dW}{d\ln b} = (\epsilon + \epsilon_\tau)W + \frac{7}{8}W^2 + \frac{1}{\pi}\bar{g}W - 2\lambda W \tag{70}$$

while

$$z - 1 = \frac{W}{8} - \frac{\bar{g}}{3\pi} \tag{71}$$

Assuming that $\bar{g}$ is a constant of order ϵ, and working with $\epsilon = \epsilon_\tau$, we can solve for the fixed points in this theory to first order in epsilon, as a function of the marginal coupling g. Fixed points are found to exist for all such values of g, and they are all found to have $z > 1$. A linear stability analysis of these fixed points demonstrates that they are stable (eigenvalues of the stability matrix are found to be complex, but with negative real parts). Thus, a stable fixed line of critical points with $z > 1$ can be obtained within this model.

In order to derive this model we have assumed that the dynamic critcal exponent satisfies $z < 2$. This allows us to retain just the frequency part in the denominator in passing from equation (41) to equation (49), which is the form used in all the subsequent analysis. In the epsilon expansion, the dynamical critical exponent differs from unity by an amount of order ϵ. Since ϵ is considered the smallest quantity in the problem, the condition on the dynamical critical exponent $z < 2$ is formally satisfied within our calculation.

These results are summarized in Figure 3, where only the couplings e^* and g for the fixed points is plotted. The $z > 1$ fixed points are found for all values of $1/g$ (which is proportional to the conductivity of the 2DEG). However, for very small values of $1/g$ they are not to be trusted, since we assume g to be $O(\epsilon)$. Therefore, these fixed points are shown with dashed lines. In the limit of very large 2DEG conductivity ($1/g$ large), the fixed point obtained is the same as for the problem with short-ranged interactions and no metallic plane, which was considered in [12] (this is made explicit in the Appendix B 4).

V. QUANTUM HALL TRANSITIONS WITH A METALLIC PLANE

In this section we consider the effect of a metallic plane on the quantum Hall transition, which, along with the superconductor to insulator transition, has been one of the most intensively studied quantum critical phenomena [24]. Most theoretical studies of the quantum Hall transition between integer quantum Hall states have used a non-interacting model of electrons in a random potential subject to a magnetic field. While it is believed that short-ranged interactions between the electrons are irrelevant [6], the same is not true of the long-ranged Coulomb interaction which is present in the physical problem of interest. It has been suggested that the non-interacting electron universality class for these quantum Hall transitions may be partially recovered by screening of the long-range Coulomb interactions by a nearby metallic gate. Increasing the screening in this way was shown experimentally to induce a change of variable-range hopping transport, away from the transition, from the Efros-Shklovksii behavior expected with Coulomb interactions to the Mott behavior expected for noninteracting electrons. [26] So one conjecture would be that a nearby metallic gate simply changes the transition to the short-range universality class.

However, as we have seen in the case of the superconductor-insulator transition, the effect of the metallic gate may well be more involved, and it cannot automatically be assumed that its sole effect is to screen the Coulomb interactions. Indeed, the general arguments in Section II B (relying as they did only on gauge invariance and the non-analytic form of the gauge field action) strongly suggest that a similar fixed line may also control the generic quantum Hall transition in the presence of a metallic plane.

To provide some calculational evidence for this suggestion, we consider a simplified model used to describe the quantum Hall transition with interacting particles, although the effects of disorder, which are believed to be crucial for the physical transition, are ignored. The model describing the critical point consists of a massless Dirac fermion coupled to a Chern-Simons gauge field [27] and interacting via the Coulomb force [14]. This theory can be used to model both integer and fractional Hall transitions by changing a statistics parameter. The Euclidean action for this model, in the absence of the metallic gate, takes the form:

$$S = S_f + S_{gauge}$$

$$S_f = \int d^2x d\tau \sum_{m=1}^{N} \bar{\psi}_m[\gamma_0(\partial_0 - \frac{i}{\sqrt{N}}a_0)$$

$$+\gamma_j(\partial_j - \frac{i\theta}{\sqrt{N}}a_j)]\psi_m \quad (72)$$

$$S_{gauge} = \int đ^2kđ\omega\, [\tilde{a}_0(k,\omega)\epsilon_{ij}k_i\tilde{a}_j(-k,-\omega) + \frac{e^{*2}}{2}\tilde{V}_{Coul}(k)|\epsilon_{ij}k_i\tilde{a}_j(k,\omega)|^2] \quad (73)$$

where the γ_μ are the Dirac matrices in three dimensional Euclidean spacetime (one representation in terms of the familiar Pauli matrices is $[\gamma_0 = i\sigma_3;\ \gamma_j = i\sigma_j]$), e^* is the Coulomb charge, θ/N the statistical angle and $\tilde{V}_{Coul}(k) = 1/k$ corresponds to a Coulomb potential of $\frac{1}{2\pi r}$. In order to exert some control over the physics of the critical point, N species of fermions were introduced. When $N = 1$ the model describes charged particles at the critical point between an insulator and a quantized Hall state with Hall conductivity $\sigma_{xy} = \frac{e^2}{h}(1 - \frac{\theta}{2\pi})$. For $\theta = 0$ this is a model of the integer transition, while $\theta = 2\pi$ describes a Mott insulator to superfluid transition. In reference [14] the model was considered in the limit of large N. The results of those investigations to order $1/N$ were as follows. The critical properties are controlled by a line of fixed points parametrized by θ (the statistics is not renormalized, as expected). For $\theta < 1.24$, the Coulomb interaction is found to be marginally irrelevant, while for $\theta > 2$, it flows to strong coupling. In between, $1.24 < \theta < 2$, the critical points are found to have a finite Coulomb charge and $z = 1$.

We now consider a system undergoing an insulator-quantized Hall state transition in the presence of a metallic 2DEG. The geometry is taken to be identical to figure 1, except that in place of the JJA, a layer undergoing the quantum Hall transition is present. The metallic 2DEG below it is assumed to have sufficiently high density of electrons that we can ignore the effect of the applied magnetic field, and treat it as a diffusive metal.

To model the quantum Hall transition, we use the anyon Mott transition model described above, but now the particles in the quantum Hall layer interact with the electrons in the metal, via the Coulomb force. Once again we integrate out the diffusive electrons in the metal to obtain an effective action for the quantum Hall transition. Again, only the part of the response that is quadratic in the gauge fields is retained, while higher order terms are neglected. It is convenient for our purposes to write down this action in a somewhat different form from that shown above. An additional temporal gauge field A_0 is introduced, and the Euclidean action can then be written as:

$$S = S_{Dirac} + S_{gauge}$$

$$S_{Dirac} = \int d^2x d\tau \sum_{m=1}^{N} \bar{\psi}_m[\gamma_0(\partial_0 - \frac{i}{\sqrt{N}}(a_0 + A_0)) + \gamma_j(\partial_j - \frac{i\theta}{\sqrt{N}}a_j)]\psi_m \quad (74)$$

$$S_{gauge} = \int đ^2kđ\omega\, [\tilde{a}_0(k,\omega)\epsilon_{ij}k_i\tilde{a}_j(-k,-\omega) + \frac{1}{2e^{*2}}\tilde{V}^{-1}_{eff}(k,\omega)|\tilde{A}_0(k,\omega)|^2] \quad (75)$$

where

$$\tilde{V}^{-1}_{eff}(k,\omega) = |k| + \sigma_e\frac{k^2}{|\omega|} \quad (76)$$

Note that gauge invariance and the non-analytic form of the terms entering the gauge part of the action (75) ensure that the statistical angle θ as well as the term containing the conductivity σ_e are not renormalized, and the Coulomb charge flows according to the equation (44). Thus, there are two strictly marginal operators parametrized by θ and σ_e/e^{*2}. We already know that the statistics parameter generates a fixed line of critical points. We would now like to address the question: under what circumstances, at a fixed value of θ, will the 2DEG also give rise to a fixed line? We will address this question within the large-N approximation. As we have seen in earlier cases, if the Coulomb interaction turns out to be marginally irrelevant, then there is a single fixed point controlling the transition (at a fixed θ) despite the fact that the term containing the 2DEG conductivity σ_e is a strictly marginal operator. This is because the gauge field A_0 decouples from the problem in this limit. We would therefore like to calculate the flow of the Coulomb charge to $O(1/N)$, in the presence of the metal. This can be done following reference [14], but with the modified action (75). For example, for the integer transition $\theta = 0$, the flow equation for the Coulomb charge in the presence of the metal can be easily worked out ($w = e^{*2}/16$):

$$\frac{dw}{d\ln b} = -\frac{8w^2}{\pi^2}\int_0^{\pi/2} d\theta \frac{\sin^2\theta}{1 + w\sin^2\theta + \sigma_e\tan\theta} \quad (77)$$

the right-hand side clearly is always negative and hence the Coulomb interaction at this integer transition is marginally irrelevant even in the presence of the metal.

The flow equations for general θ and σ_e are complicated and we do not attempt to write them out here. Instead we will argue that at least for small enough σ_e there is a range of θ fixed points for which, if we fix θ, a fixed line parametrized by σ_e can be obtained. First consider the case of $\sigma_e = 0$ where a finite value of the Coulomb charge was found for fixed points in a certain range: $1.24 < \theta < 2$ [14]. Imagine fixing a particular value of θ inside this range. Then, for sufficiently small $\sigma_e > 0$, a fixed point with finite Coulomb charge is still to be expected. The strictly marginal operator contained in the σ_e term will affect the universal properties of the fixed point by inducing a fixed line at this particular value of θ. Since the renormalized Coulomb charge is expected to remain finite along this fixed line, $z = 1$ will obtain. In addition to these fixed points, we can consider the effect

of the metal (with σ_e small) on the fixed points for $\theta > 2$. There, the $e^* \to \infty$ behaviour in the model without the metallic plane, is expected, in the presence of a metallic plane, to lead to a line of $z > 1$ fixed points. These fixed points should also be accessible within the large N technique; although the Coulomb charge flows to infinity the gauge field fluctuations are small and controlled by the marginal coupling $e^{*2}/\sigma_e N$.

Thus, in this model of quantum Hall transitions, as in the superconductor-insulator transitions discussed previously, the proximity of the 2DEG can give rise to a line of fixed points controlling the critical properties and hence affect the phase transition in a non-trivial fashion.

VI. COMMENTS ON LOCAL DISSIPATION MODELS

In this section, we briefly comment on previous theoretical studies of Josephson arrays with "local dissipative baths" whose results bear superficial similarities with those in the present paper. It is important to first realize that we have focused on a physical situation where the origin of the dissipation is very clear. It is due to the coupling between the Cooper pairs and the gapless diffusive electrons in the metal. In contrast, Ref. [9] postulated the presence of dissipation due to some unspecified local degrees of freedom. That paper also suggested a fixed line controlling the transition between the superconducting and insulating phases, once the dissipation exceeds a critical strength $\alpha_0 = 2/3$. The dissipative term for each superconducting grain is assumed to be

$$S_d = \alpha \int d\tau\, d\tau' \frac{(\phi(\tau) - \phi(\tau'))^2}{|\tau - \tau'|^2}. \tag{78}$$

This model is not expected to describe the experiments in Refs. [4,5]. The zero diffusion constant limit of our model resembles superficially the local dissipation models considered in [18], which in turn differs from (78) in that the local dissipation is coupled to phase differences across junctions. Moreover, in our case the crucial boundary condition on the phase in the imaginary time direction (38) can differ from that derived for more phenomenological models of local dissipation [18,19]. Furthermore, the presence of long ranged interactions is crucial to the physics discussed in this paper.

Finally we point out a problem with the approximation used in [9] to derive the presence of a fixed line in the absence of both long-ranged interactions and disorder. We show in Appendix C, that when exactly the same approximation is applied to the 3D anisotropic XY model, one erraneously concludes the existence of a fixed line in that problem, in which the critical properties are actually controlled by a single fixed point.

VII. IMPLICATIONS FOR EXPERIMENTS

In this section we will consider the physical consequences of the field theoretical results derived earlier. We first address a quantitative question - at what value of the 2DEG conductivity are the critical properties of the superconductor-insulator transition significantly affected? For the $z = 1$ fixed points, this will clearly occur when the value of the marginal coupling $(1/g)$ is of the same order or larger than the renormalized Coulomb coupling $(1/e_c^{*2})$. Reinstating the physical parameters, this yields the following condition on the 2DEG conductivity in SI units: $\sigma_e^{phys} > (\frac{e^{*2}}{e_c^{*2}}) 2 c_s \kappa\epsilon_0$, where $\kappa\epsilon_0$ is the permittivity constant and c_s is the bare speed of sound in the superfluid (in the absence of Coulomb interactions). If we make the naive assumption that the renormalized Coulomb force is roughly the same as the bare Coulomb force $e_c^* \approx e^*$, then we can obtain an estimate for the 2DEG conductivity at which a significant effect on the critical properties is felt. The speed of sound is given by $(c_s \sim \frac{a}{\hbar}\sqrt{E_J E_C})$, where a is the distance between superconducting grains and E_J and E_C are the Josephson coupling energy and the charging energy of a grain. Typically, at the transition $E_J \sim E_C$ (if the 2DEG conductivity is not too large) and so we obtain $c_s \sim E_C a/\hbar$. Finally, substituting the form of the charging energy for a grain of radius R, $E_C = e^{*2}/4\pi\kappa\epsilon_0 R$, we obtain that the nondimensional conductivity should be $2c\kappa\epsilon_0 \sim \frac{e^{*2}}{h}(a/R)$. This is roughly of order the universal conductivity, if the grain size is approximately the same as the distance between grains. Hence, we expect the effect of the 2DEG to be substantial once its conductivity is of order $(6.4\text{k}\Omega)^{-1}$. This regime is readily accessible in experiments.

An alternate estimate of this conductivity for the $z = 1$ fixed line can be derived from the epsilon expansion solution to the model with particle-hole symmetric disorder. There, we know that the exponents of the $z = 1$ fixed points are strongly affected once the marginal coupling is of the order of $1/g_c = 1.7 \times 10^{-3}\epsilon$. Setting $\epsilon = 1$ and using the relation $\sigma_e^{phys} = (2\pi)\frac{e^{*2}}{h}\frac{1}{g}$, we obtain $\sigma_e^{phys} = 0.01\frac{e^{*2}}{h} \approx (600k\Omega)^{-1}$. This corresponds to a smaller value of the conductivity than the earlier estimate, because the Coulomb charge at the fixed point is strongly renormalized. For the $z > 1$ fixed points, the marginal coupling controls the gauge field fluctuations. However, once the 2DEG conductivity greatly exceeds the quantum of counductance $1/g \gg 1 \Rightarrow \sigma_e \gg (2\pi)\frac{e^{*2}}{h} = (1.1k\Omega)^{-1}$ the critical exponents approach those of the short-ranged model without a metallic plane.

We now briefly consider how the results for the fixed line of critical points obtained in this paper may be tested experimentally, both for the superconductor-insulator transition as well as the quantum Hall transition. Ideally, one would like to investigate the universal critical

properties at the transition, such as the critical exponents and the conductivity at the transition, as a function of the 2DEG (metallic ground plane) conductivity. A fixed line of critical points implies that these properties are a function of where the phase boundary is crossed. Although these various 'universal' quantities would vary along the fixed line, there would be universal relations amongst them, since they can all be expressed as a function of a single parameter, the strength of the marginal coupling. We also note the caveat, that for large values of the 2DEG conductivity, there will be region about the critical point where the system will behave like the short-ranged interaction model, before ultimately crossing over to the true fixed point behaviour very close to the transition.

Finally, we note that in a finite sized system (such as the JJA in reference [4] which is a 40x40 system) the transition is rounded off when the correlation length exceeds the linear size of the system. The width of this region (Δg), in terms of a control parameter g that tunes the transition, is given by the exponent ν ($\Delta g \sim L^{-1/\nu}$, where L is the linear system size). The double epsilon expansion described earlier, on a model with particle-hole symmetric disorder that exhibits the superconductor to insulator transition, revealed a strong variation of ν along the fixed line. Such a variation could presumably be detected by studying the width of the transition in these finite sized systems.

VIII. CONCLUSIONS

In this paper, we have studied the effects of a proximate metallic plane on various two dimensional localization transitions with an emphasis on the superconductor-insulator transition. Such a metallic plane has two principal effects—it provides a mechanism for screening the long-ranged Coulomb interaction, and it is a source of dissipation due to the gapless diffusive electrons. The interplay of these two effects leads to interesting physical phenomena. Perhaps the most interesting result in this paper is the possibility of a fixed line with variable critical exponents controlling the transition. In addition, right at the transition, a temperature independent conductivity that will vary along the fixed line is expected.

Our results are of direct relevance to experiments probing the superconductor-insulator transition in Josephson junction arrays in the presence of a metallic plane [4]. So far these experiments have not probed the universal scaling regime near the transition. We hope that our results will focus future experimental work on this regime.

A similar scenario may be expected to hold for quantum Hall transitions in the presence of a proximate metallic plane. A line of fixed points would again imply variable critical exponents and a temperature independent value for the diagonal conductivity at the transition. Recently, quantum Hall systems with nearby metallic planes have been prepared [26] and it would be interesting to experimentally investigate the effect of the metallic plane on the critical properties at the quantum Hall transitions.

One limitation of our field theoretic approach is that, following Hertz [17], we have integrated out the gapless diffusive electrons to obtain an effective action for the critical bosonic Cooper pair degrees of freedom. As with other problems involving quantum phase transitions in fermionic systems, a more satisfying theoretical approach would keep both gapless fermionic and bosonic modes as part of the effective theory and treat them on equal footing. Unfortunately, such an approach is not available at present.

Another assumption that has been made which is needed to justify an approximation used through this paper is that the fixed points satisfy $z < 2$. We leave it to future work to investigate whether superconductor to insulator transitions in the presence of Coulomb interactions and a metal can be controlled by fixed points with dynamical exponents $z \geq 2$, and if so what their description is.

We conclude by noting that our results show that even a simple Coulomb interaction between the Cooper pairs and the gapless fermionic degrees of freedom has a profound effect on the universality class of the superconductor-insulator transition. Should the fixed line found in this paper survive the inclusion of processes where the Cooper pair can decay into two electrons, it will lead to a temperature independent though apparently non-universal conductivity at the superconductor-metal transition.

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APPENDIX A: LARGE N CALCULATION OF $Z-1$ IN THE CLEAN CASE.

In this section we describe in some detail how the dynamical critical exponent z can be calculated within the large-N approximation, in the absence of disorder.

We consider the problem directly in $D=2$, but with N species of bosons coupled to the scalar component of a gauge field as described by equation (43). The bare propagators for the gauge field and the bosons are:

$$\mathcal{G}^0_A(k,\omega) = \frac{e^{*2}}{N}(|k| + \sigma_e \frac{k^2}{|\omega|})^{-1} \tag{A1}$$
$$\mathcal{G}^0(k,\omega) = (q^2+\omega^2)^{-1} \tag{A2}$$

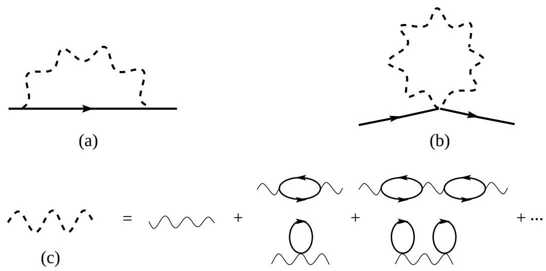

FIG. 5. To evaluate the O(1/N) correction to the dynamical critical exponent; only diagram (a) needs to be considered, where the solid line is the boson propagator at criticality and the dashed-wavy line is the RPA form of the gauge propagator, defined in (c), that appears to this order in the large-N approximation.

Since it is only the dynamics and coupling of the gauge field that distinguishes space and time, any contribution to $z-1$ must come entirely from this interaction. We have already noted in Section III B 1 that to leading order in 1/N (saddle point) the gauge field decouples from the problem, and the dynamical critical exponent stays equal to unity at this stage. Therefore, we calculate this quantity to O(1/N). First, we recognize that the gauge field propagator will be dressed by boson density fluctuations as shown in Figure 5a, so that we obtain the RPA form:

$$\mathcal{G}^{-1}_A(k,\omega) = [\mathcal{G}^0_A(k,\omega)]^{-1} + \Pi^0(k,\omega) \tag{A3}$$

where Π^0 is the contribution from the pair of boson bubble diagrams in Figure 5c

$$\Pi^0(k,\omega) = \int \bar{d}^2K \bar{d}\Omega\, \mathcal{G}_0(K,\Omega) \\ [(\omega+2\Omega)^2\mathcal{G}_0(k+K,\omega+\Omega) - 1] \tag{A4}$$
$$= \frac{1}{16}\frac{k^2}{\sqrt{k^2+\omega^2}} \tag{A5}$$

In order to calculate $(z-1)$ to O(1/N), we need to evaluate the self energy contribution arising from the diagrams in Figure 5b. We will be interested in isolating terms in the self energy that are of the form:

$$\Sigma(k,\omega) \sim \eta k^2 \ln(\sqrt{k^2+\omega^2}/\Lambda) + \eta_\omega \omega^2 \ln(\sqrt{k^2+\omega^2}/\Lambda) \tag{A6}$$

where Λ is the ultraviolet cutoff. These contributions will modify the boson propagator since:

$$\mathcal{G}^{-1}(k,\omega) = [\mathcal{G}^0(k,\omega)]^{-1} - \Sigma(k,\omega) \tag{A7}$$

and thus affect the power laws that characterise the boson correlators at criticality. The dynamical critical exponent is then given by:

$$z - 1 = \frac{1}{2}(\eta_\omega - \eta) \tag{A8}$$

Notice, that since we are only interested in contributions of the form (A6), which vanish in the zero frequency limit, we only need to calculate $\Sigma(k,\omega) - \Sigma(0,0)$, therefore the second diagram in Figure 5b, which gives a momentum independent contribution, can be ignored. Evaluating the resulting diagram:

$$\Sigma(k,\omega) - \Sigma(0,0) = \\ \frac{1}{N}\int \bar{d}^2K\bar{d}\Omega\, [(2\omega+\Omega)^2\mathcal{G}^0(k+K,\omega+\Omega) \\ -\Omega^2\mathcal{G}^0(K,\Omega)]\mathcal{G}_A(K,\Omega) \tag{A9}$$

we can obtain the correction to the dynamical critical exponent to O(1/N), whic is:

$$z - 1 = \frac{1}{N} I(e^{*2},\sigma_e)$$
$$I(e^{*2},\sigma_e) = \int_0^{\pi/2} \frac{d\theta}{2\pi^2}\frac{\sin^2\theta(1-3\sin^2\theta)}{e^{*-2}(1+\sigma_e\tan\theta) + \frac{\sin\theta}{16}}$$

Note, that this can be shown to imply that $z-1>0$ for all $e^*>0$, $\sigma_e>0$ and hence the only fixed point is at $e^*=0$.

APPENDIX B: THE ϵ, ϵ_τ EXPANSION

We consider the problem in $D = 3-\epsilon$ spatial dimensions, with particle-hole symmetric disorder correlated in ϵ_τ dimensions, given by the action in equation (55). We consider integrating out all modes with wavevectors in the range $[\Lambda/b, \Lambda]$. The additional terms generated lead to a renormalization of the terms in the action; the quadratic derivative terms are now multiplied by the scale factors Z_ω, Z_k and the other terms acquire factors Z_r, Z_u and $Z_{\bar{W}}$. Note, in dimensions $D<3$, the Coulomb term, as well as the term arising from coupling to the 2DEG in the gauge propagator are unchanged on integrating out high frequency modes since they are non-analytic in frequency wave-vector space. On rescaling the momenta $k' = bk$ to restore the cutoff to Λ and also rescaling $\omega' = b^z\omega$ and the fields $\psi' = Z_k^{\frac{1}{2}}\psi$ and $\tilde{A}'_0 = b^{-D}\tilde{A}_0$ (the gauge field scaling follows from gauge invariance) to obtain a new set of couplings at this scale:

$$u' = b^{\epsilon+1-z} Z_k^{-2} Z_u u \tag{B1}$$
$$\bar{W}' = b^{\epsilon+1-z+\epsilon_\tau} Z_k^{-2} Z_{\bar{W}} \bar{W} \tag{B2}$$
$$e^{*'2} = b^{z-1} e^{*2} \tag{B3}$$
$$r' = b^2 Z_k^{-1} Z_r r \tag{B4}$$

In the following, we will look for fixed point solutions to the above flow equations, assuming that ϵ, ϵ_τ are small. In that limit, we are justified in looking for fixed points where the couplings are small, and hence a perturbative evaluation of the flows is feasible.

1. Calculation of z and η; Flow of e^*.

Here we calculate the change in the quadratic derivative terms (Z_k, Z_ω) on integrating out the high momentum modes. In practice, this is most easily done by integrating all modes below a cutoff Λ, and extracting the log divergent contributions to the boson propagator $\mathcal{G}$:

$$\mathcal{G}_{ren}^{-1}(k,\omega) = k^2 + \omega^2 - \eta k^2 \ln(\sqrt{k^2+\omega^2}/\Lambda) - \eta_\omega \omega^2 \ln(\sqrt{k^2+\omega^2}/\Lambda) \tag{B5}$$

More details on how these two proceedures are related to each other may be found in [23]. Here, we just note that if integrating all modes below the cutoff Λ gives rise to the additional term in equation (B5), then the contribution on just integrating modes in a shell betwen $[\Lambda/b, \Lambda]$, can be found by subtracting the same term but with $\Lambda \to \Lambda/b$ which yields:

$$(\eta k^2 + \eta_\omega \omega^2) \ln b$$

. These can then be related to the quantities we want to calculate:

$$Z_k = 1 + \eta \ln b \tag{B6}$$
$$Z_\omega = 1 + \eta_\omega \ln b \tag{B7}$$

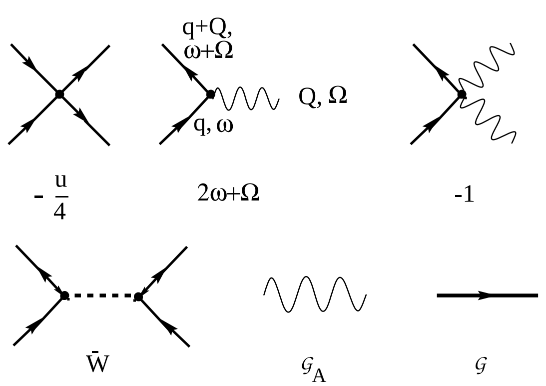

FIG. 6. Feynman rules used in the ϵ, ϵ_τ expansion. The vertices arising from the $|\psi|^4$ interaction, the gauge interaction and disorder generated interaction are shown. The dashed disorder line only transfers the $D+1-\epsilon_\tau$ spatial components of the momentum.

The Feynman rules derived from the action (55) are shown in Figure 6, where the boson propagator at criticality is given by $\mathcal{G}^{-1} = (k^2+\omega^2)$ and the gauge propagator is given by $\mathcal{G}_A^{-1} = e^{*-2}(k^{D-1} + \sigma_e \frac{k^D}{|\omega|})$.

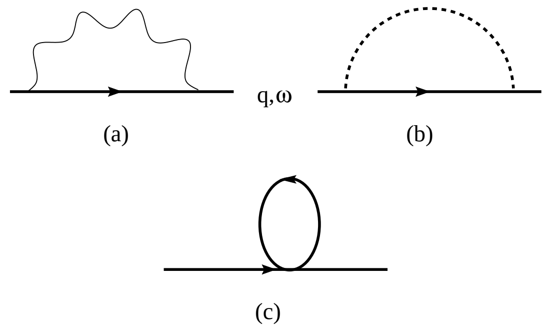

FIG. 7. The diagrams (a) and (b) contribute to renormalizing the the quadratic derivative terms in the action, while all three diagrams contribute to the renormalization of the quadratic term r.

We are now in a position to calculate the quantities in (B7). Since we are interested in the momentum dependent contribution to the self energy, we only need to calculate $\Sigma(k,\omega) - \Sigma(0,0)$. The two diagrams of interest then are shown in Figure 7a and 7b. We work to lowest order in ϵ and therefore the integrals will be performed in $D = 3$, and the terms that depend on the logarithm of the cutoff will be extracted. The contribution from gauge fluctations (Figure 7a) is:

$$\Sigma(k,\omega) - \Sigma(0,0) = \int đ^3K đ\Omega\, \mathcal{G}_A(K,\Omega) [(2\omega+\Omega)^2 \mathcal{G}(k+K,\omega+\Omega) - \Omega^2 \mathcal{G}(K,\Omega)] \tag{B8}$$

which yields:

$$\eta_\omega^{A_0} = -\frac{e^{*2}}{2\pi^2} \int_0^{\frac{\pi}{2}} \frac{d\theta}{\pi} \frac{4 - 9\cos^2\theta + 4\cos^4\theta}{1+\sigma_e \tan\theta} \tag{B9}$$
$$\eta^{A_0} = \frac{e^{*2}}{6\pi^2} \int_0^{\frac{\pi}{2}} \frac{d\theta}{\pi} \frac{(4\cos^2\theta - 1)\cos^2\theta}{1+\sigma_e \tan\theta} \tag{B10}$$

Similarly, the contribution arising from the disorder diagram in Figure 7b can be calculated. Since the disorder induced interaction is independent of spatial momentum transfer, $\Sigma(q,0) = \Sigma(0,0)$, the disorder contribution to η vanishes. However, at finite frequency transfer we have:

$$\Sigma(0,\omega) - \Sigma(0,0) = -\omega^2 \bar{W} \int \frac{đ^{D+1-\epsilon_\tau} Q}{Q^4} \tag{B11}$$

which, to lowest order in ϵ_τ can be evaluated to give

$$\eta_\omega^{\bar{W}} = -\frac{\bar{W}}{8\pi^2} \tag{B12}$$

putting this all together we have

$$\eta = \eta^{A_0}$$
$$\eta_\omega = \eta_\omega^{A_0} + \eta_\omega^{\bar{W}}$$

from this the flow equation for the Coulomb charge is easily derived:

$$\beta(e^*) = \frac{de^{*2}}{d\ln b} = (z-1)e^{*2} \tag{B13}$$

$$z - 1 = \frac{1}{2}(\eta_\omega - \eta) \tag{B14}$$

after redifining the couplings $q^2 = \frac{e^{*2}}{2\pi^2}$ and $W = \frac{\bar{W}}{2\pi^2}$ we get equation (58).

2. Flow Equations for u:

We begin by calculating the renormalization of the quartic term (u) arising from the integration of high frequency modes ($Z_u u$). Diagrams Figure 8a-g contribute, and we can write the result as $Z_u u = u+$(Fig. 8a+..+g)$\ln b$. We will evaluate these diagrams and extract the log divergent parts; the external lines are assumed to carry zero momentum.

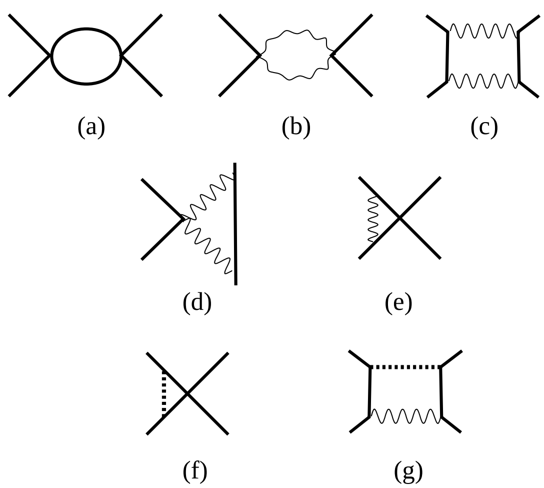

FIG. 8. The diagrams contributing to the renormalization of the quartic interaction.

The contribution from the first diagram can be found in any standard reference on the ϵ expansion for short-ranged model [23], and is given by:

$$8\text{a} \to -\frac{5}{2}\frac{u^2}{8\pi^2}$$

Next, contributions arising from the gauge interaction (Figure 8b-e) can be evaluated. Details about the combinatorial factors that appear for these diagrams are not repeated here as they as they are discussed in [22]; the only difference for our calculations is that the gauge propagator is modified due to coupling to the 2DEG. The result for these diagrams then is

$$8\text{b}+8\text{c}+8\text{d} \to -\frac{e^{*4}}{2\pi^2}[\frac{2}{\pi}\int_0^{\pi/2} d\theta \frac{\sin^2\theta}{(1+\sigma_e\tan\theta)^2}]$$

while the graph in Figure 8e contributes:

$$8\text{e} \to \frac{ue^{*2}}{4\pi^2}[\frac{4}{\pi}\int_0^{\pi/2} d\theta \frac{\cos^2\theta}{1+\sigma_e\tan\theta}]$$

The diagram in Figure 8f which involves the disorder can be computed in the following way. There are ${}^4C_2$ diagrams of the type shown, and hence the contribution is:

$$8\text{f} = {}^4C_2 u\bar{W}\int \frac{d\!\!{}^-{}^4p}{p^4} \to \frac{3u\bar{W}}{4\pi^2}$$

Finally, the diagram in Figure 8g makes no contribution. Since the external lines are at zero frequency, and the disorder interacton, being independent of time, does not change the frequency, the coupling to the gauge field vanishes.

Putting together these results, as well as the factors arising from rescaling the fields (B1, B6, B14) and redefining the couplings as in (57), we obtain the flow equation (59).

3. Flow Equations for Disorder Coupling

We first consider the renormalization of the disorder term ($\bar{W}$) arising from the integration of high frequency modes ($Z_{\bar{W}}\bar{W}$). Diagrams Figure 9a-d contribute, and we can write the result as $Z_{\bar{W}}\bar{W} = \bar{W}+$(Fig. 9a+..+d)$\ln b$.

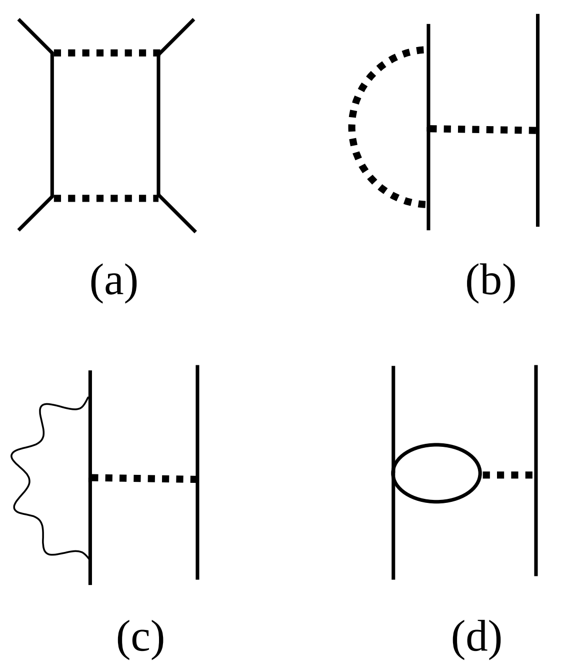

FIG. 9. The diagrams contributing to the renormalization of the interaction induced by disorder.

To evaluate the diagram in Figure 9a we note there are two identical processes that contribute, with the boson lines parallel or anti-parallel. Similarly, there are two equivalent processes associated with the diagram in Figure 9b, with the disorder induced interaction decorating either the left or right boson line. These diagrams all make the same contribution; putting them together:

$$9\text{a}+9\text{b} = 4\bar{W}^2 \int \frac{đ^4 p}{p^4} \rightarrow \frac{\bar{W}^2}{2\pi^2}$$

Similarly, for the diagram in Figure 9c there are two equivalent contributions, since the gauge interaction can decorate either the left or the right boson line. Hence,

$$\begin{aligned} 9\text{c} &= 2\bar{W} \int đ^3 K đ\Omega\, \Omega^2 \mathcal{G}_A(K,\Omega)\mathcal{G}^2(K,\Omega) \\ &\rightarrow \frac{e^{*2}\bar{W}}{4\pi^2}[\frac{4}{\pi}\int_0^{\pi/2} d\theta \frac{\cos^2\theta}{1+\sigma_e \tan\theta}] \end{aligned}$$

Finally, the the diagram in Figure 9d also has two equivalent realizations, and the boson self interaction loop can be inserted in four ways, which yields:

$$9\text{d} = 8\bar{W}(-u/4)\int \frac{đ^4 p}{p^4} \rightarrow -\bar{W}\frac{u}{4\pi^2}$$

Putting together these results, as well as the factors arising from rescaling the fields (B2, B6, B14) and redefining the couplings as in (57), we obtain the flow equation for the disorder coupling (60). This completes the derivation of the flow equations (58 - 60).

4. The $z > 1$ Fixed Line

The fixed points in the presence of a metallic plane in the case $e^* \rightarrow \infty$ are obtained from the equations (69, 70) assuming that the marginal coupling $\bar{g}$ is $O(\epsilon)$. This yields the fixed point values of the couplings, (in terms of $x(\bar{g}) = \frac{1}{3}(\frac{4}{\pi}\bar{g} + 4\epsilon + 11\epsilon_\tau)$):

$$\lambda_c = \frac{1}{3}(x(\bar{g}) + \sqrt{\frac{7}{2}\bar{g}^2 + x^2(\bar{g})}) \tag{B15}$$

$$W_c = \frac{8}{7}(2\lambda_c - \frac{\bar{g}}{\pi} - \epsilon - \epsilon_\tau) \tag{B16}$$

the couplings are $O(\epsilon)$ and the dynamical critical exponent is given by:

$$z = 1 + \frac{W_c}{8} - \frac{\bar{g}}{3\pi} \tag{B17}$$

These solutions exist for all $O(\epsilon)$ values of $\bar{g}$, and have $z > 1$, which ensures that $e^* = \infty$ does not flow. In the linit $\bar{g} \rightarrow 0$ (which corresponds to a large value of the 2DEG conductivity) we recover the short-range interacting boson case, in the absence of a metallic plane (studied in [12]). Note, the flow equations (69, 70 with $\bar{g} = 0$) however differ from those in [12]. This difference is a result of treating the time direction as distinct in the presence of Coulomb interactions [13], which leads us to scale the measure $[d\omega dk^D]$ as L^{-z-D} rather than as $L^{-\epsilon_\tau(z-1)-D}$ as in [12] where only the ϵ_τ dimensions are assumed to scale with the dynamical exponent. Therefore when we continue the fixed point obtained in the presence of Coulomb interactions to the $\bar{g} = 0$ fixed point, it differs in details from the result obtained in [12] but agrees with the short-ranged fixed point quoted in [13].

5. The Critical Exponent ν

To calculate the exponent ν, we need the flow equations for the coefficient of the quadratic term (r). In fact, we only need those terms in the flow equations that themselves contain r. The diagrams that then need to be considered are those in Figure 7a,b and c, evaluated with zero momentum and frequency in the external lines. In these, the terms proportional to r that contribute to its renormalization are:

$$\begin{aligned} 7\text{a} &= r\int đ^3 K đ\Omega\, \mathcal{G}^2(K,\Omega)\Omega^2\mathcal{G}_A(K,\Omega) \\ &\rightarrow r\frac{e^{*2}}{2\pi^2}[\frac{4}{\pi}\int_0^{\frac{\pi}{2}} d\theta\frac{\cos^2\theta}{1+\sigma_e\tan\theta}] \\ 7\text{b} &= r\bar{W}\int đ^4 p \mathcal{G}^2(p) \\ &\rightarrow r\frac{\bar{W}}{8\pi^2} \end{aligned}$$

finally, evaluating the diagram in Figure 7c, we note that there are four ways to pick boson lines to make a loop and hence:

$$\begin{aligned} 7\text{c} &= 4r(-\frac{u}{4})\int đ^4 p\mathcal{G}^2(p) \\ &\rightarrow -r\frac{u}{8\pi^2} \end{aligned}$$

Combining this with the effect of rescaling the fields, as given in equation (B4) and rescaling the couplings according to (57), we obtain the following flow equation for this relevant coupling.

$$\frac{dr}{d\ln b} = [2 - \lambda + \frac{W}{4} + \frac{D}{12}q^2]r + \ldots \text{(terms independent of r)} \tag{B18}$$

Where the coefficient D is a function of σ_e and is defined in equation (67). At the fixed point this leads to the following equation for the critical exponent ν:

$$\nu^{-1} = 2 - \lambda_c + \frac{W_c}{4} + \frac{D}{12} q_c^2 \tag{B19}$$

since these fixed point couplings are all small [O(ϵ)], we can invert this expression to obtain equation (66). In the limit $\sigma_e = 0$, we can compare this answer with earlier work. The coefficient of the q_c^2 term differs from the expression in [13], but agrees with that in [22]

The Exponent ν for the $z > 1$ Fixed Line

For this fixed line, ν can be derived from equation (66) by formally letting $q^2, \sigma_e \to \infty$ but holding the ratio $q^2/\sigma_e = \bar{g}$ finite. This yields:

$$\nu = \frac{1}{2} + \frac{1}{4}(\lambda_c(\bar{g}) - \frac{\bar{g}}{3\pi} - \frac{W_c}{4}(\bar{g})) \tag{B20}$$

6. Stability of the Fixed Points:

We would like to make sure that besides the relevant variable r, the fixed point is stable to perturbing away in any of the other couplings. A linear stability analysis can be performed on the fixed point at (q_c^2, W_c, λ_c) by move slightly away ($q_c^2 + \delta q^2$, $W_c + \delta W$, $\lambda_c + \delta\lambda$) and asking how the deviations evolve under resclaing. This leads to the equation,

$$\frac{d}{d\ln b} \begin{pmatrix} \delta q^2 \\ \delta W \\ \delta\lambda \end{pmatrix} = \mathbf{M_c} \begin{pmatrix} \delta q^2 \\ \delta W \\ \delta\lambda \end{pmatrix} \tag{B21}$$

where

$$\mathbf{M_c} = \begin{bmatrix} \partial_{q^2}\beta(q^2) & \partial_W\beta(q^2) & \partial_\lambda\beta(q^2) \\ \partial_{q^2}\beta(W) & \partial_W\beta(W) & \partial_\lambda\beta(W) \\ \partial_{q^2}\beta(\lambda) & \partial_W\beta(\lambda) & \partial_\lambda\beta(\lambda) \end{bmatrix} \tag{B22}$$

where the derivatives of the beta functions are evaluated at the fixed point. If the eigenvalues of $\mathbf{M_c}$ are all negative, then the perturbation will die out on running the RG, and the fixed point is stable.

Stability of $z = 1$ Fixed Points

We have numerically evaluated these eigenvalues along the $z = 1$ fixed line parametrized by $0 < \sigma_e < 1.99$, and find that all these critical points are stable (we have assumed $\epsilon = \epsilon_\tau$ which is consistent with the limit of $\epsilon = 1$, $\epsilon_\tau = 1$ that we are finally interested in). A pair of the eigenvalues, it turns out, is complex (and conjugate to each other), but they have negative real parts. The least negative eigenvalue ranges from $\sim -0.5\epsilon$ (at $\sigma_e = 0$) to $\sim -0.3\epsilon$ (near $\sigma_e = 1.99$).

Stability of $z > 1$ Fixed Points Here, the stability matrix we need to consider is just a 2×2 matrix since the condition $z > 1$ which is satisfied by these fixed points ensures $1/e^* = 0$ remains stable. The eigenvalues of this matrix were evaluated numerically (assuming $\epsilon = \epsilon_\tau$) for the full range of g (assumed $O(\epsilon)$) and were found to be complex but with negative real parts. Thus, all these points on the $z > 1$ fixed line are found to be stable.

APPENDIX C: THE ANISOTROPIC XY MODEL

This appendix shows that the arguments in [9] for a fixed line for the dissipation term

$$S_d = \alpha \int d\tau\, d\tau' \frac{(\phi(\tau) - \phi(\tau'))^2}{|\tau - \tau'|^2}, \tag{C1}$$

which unlike in our model do not depend on either long-ranged interactions or disorder, predict incorrect behavior in a simpler case and hence are unreliable compared to controlled approximations such as the ϵ-expansion.

The approach of [9] predicts a continuous set of transitions for a simple related problem, an anisotropic XY model, where only one superconducting transition is present. This suggests that phase-squared dissipation likely introduces at most one new universality class for the superconducting transition. The subtle problem in the derivation of the effective action in [9] is that only the leading (two-spin) term is kept in the Hubbard-Stratonovich transformation used to decouple the junctions.

Consider a classical 3D system made of layered 2D XY models with interplane coupling $J_z \cos(\theta_i - \theta_{i+1})$, where i is a layer index. The action after taking the continuum limit in the planes is

$$S_{\rm XY} = \sum_i \int d^2x\, [J_{xy}(\nabla\theta_i)^2 - J_z\cos(\theta_i - \theta_{i+1})]. \tag{C2}$$

Here the couplings are defined to include $\beta = 1/kT$. If $J_z = 0$, then the system can have algebraic in-plane order for J_{xy} above the Kosterlitz-Thouless transition. In this algebraically ordered phase, the spin-spin correlation $\langle S_{\mathbf{r_1}} \cdot S_{\mathbf{r_2}}\rangle \propto |\mathbf{r_1} - \mathbf{r_2}|^{-\alpha}$, for some value $0 \le \alpha \le 1/4$ depending on the in-plane coupling. Introducing a complex boson field $\psi \equiv S_x + iS_y$ to decouple the planes,

$$\begin{aligned} S = \sum_i \int d^2x\, [&J_{xy}(\nabla\theta_i)^2 + |\psi_i - e^{i\theta_i}|^2 \\ &- J_z(\psi_i^*\psi_{i+1} + \psi_{i+1}^*\psi_i)/2]. \end{aligned} \tag{C3}$$

Now the approach of [9] is based on an effective action for the full 3D model with nonzero interplane coupling J_z, using the above 2D correlations. The quadratic part of this action reads in cylindrical coordinates

$$S_{\rm eff} = \int k\, dk\, d\phi\, dk_z\, ({k_z}^2 + k^{\alpha-2} + m^2)|\psi(k,\phi,k_z)|^2. \tag{C4}$$

Now this quadratic part is used in [9] to calculate the critical conductivity as support for the claim of a continous set of universality classes.

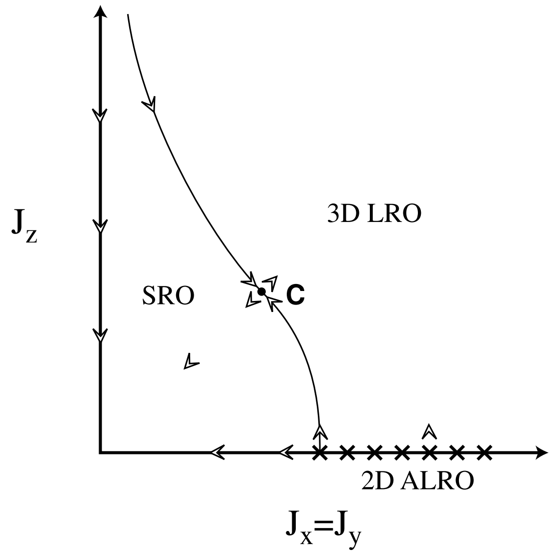

FIG. 10. Phase diagram of 3D XY model with direction-dependent couplings $J_z \neq J_x = J_y$. Arrows indicate direction of projected RG flows. Despite the line of fixed points for $J_z = 0$, all transitions between the long-range ordered (superconducting) phase and short-ranged (insulating) phase are controlled by the single critical point **C**.

Even though (C4) reproduces the in-plane two-spin correlation function in the absence of interplane coupling, it cannot be used at this level of approximation to study the transition into the superconductor. The actual phase diagram of the anisotropic XY model is shown in Fig. 10. There is a single fixed point that controls the superconducting transition for all nonzero values of the couplings, with unique values of the critical exponents.

The above suggests that the model considered in [9] may not in fact have a continuous set of superconducting transitions once $\alpha > \alpha_0 = 2/3$. The dissipation generates a series of of terms in the ψ action; all of these are irrelevant for small α, but the higher-order terms become relevant at the same time as the leading term. Weak phase-squared dissipation is indeed irrelevant, but it is not clear what happens once the dissipation is relevant.

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